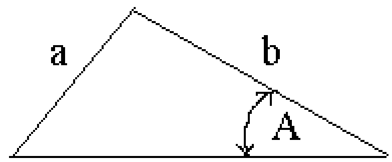
FINALS I 1981-82

1. It is well known that two sides and the included angle uniquely determine a triangle. Describe completely the relationships among sides a,b and $\angle A$ (where a is the side opposite $\angle A$ and b forms a side of A) so that these determine exactly one triangle.



2. Given the pair of equations $x^2 + y^2 = 1$ and $(x - 2)^2 + (y - 3)^2 = r^2$, find all values of r, if any, for which there is exactly one solution x,y.

3. Suppose that p is a prime number. Show that there are no integers x and y for which $x^4 - y^4 = p$.

4. A monkey tries to climb a slippery 10 foot high pole. Starting at the bottom, he climbs 5 feet and slides half way to the ground. He then climbs an additional 5 feet and again slides half-way to the ground. This process is repeated indefinitely unless the monkey reaches the top of the pole.

(a) How high is the monkey after the n^{th} climb (and before sliding)? Simplify your answer.

(b) Using (a), determine whether the monkey ever reaches the top.

(c) Try to answer (b) without using without using any formula such as in (a).

5. Given that the 3 roots of the polynomial $36x^3 + ax^2 + bx + 2$ are in the ratios 1:3:4 find the roots and the coefficients a and b.

6. Suppose m is a positive integer. Show that there is a unique sequence of integers $a_1, a_2, a_3, \ldots$, $0 \leq a_j \leq j$, such that

$$m = a_1 1! + a_2 2! + a_3 3! + \ldots \text{ where } n! = 1\text{x}2\text{x}3\text{x}\ldots\text{x}n.$$

Of course eventually the a_j are 0, so the sum is really a finite one.

. 7. There are 4 men, a monkey, and a pile of coconuts on an island. The first man divides the coconuts into 4 equal piles, finding one left over which he gives the monkey, and then takes all of the coconuts in one of the piles, leaving the rest. Later the second man arranges the remaining coconuts into 4 equal piles, leaving one left over which he gives the monkey, and then takes all of the coconuts in one of the piles and leaves the rest. Later, the third man does the same, and finally so does the fourth, each time taking one of 4 equal piles with 1 coconut left over going to the monkey. If there were fewer than 500 coconuts originally, how many coconuts were there?