FINALS XII 1992-93

1.(a) A person invests P = \$1000 at a rate r = 8% of interest compounded n = 2 times per year. What rate s of interest not compounded would produce the same interest in one year?

(b) In (a) let P,n,r be arbitrary and find a formula for s in terms of P,n,r.

(c) Suppose \$100 is invested at a rate r% interest compounded 4 times per year for 2 years. Find the value of r if the final value of the investment is \$110. (Express answer to 2 decimal places).

(d) Suppose \$$P$ is invested at a rate r% compounded n times per year for k years. Find the value of r in terms of n and k if the final value of the investment is \$(2P).

2. A total of $n^2 + 2n + 29$ pieces of candy is divided equally among 15 girls and n boys.

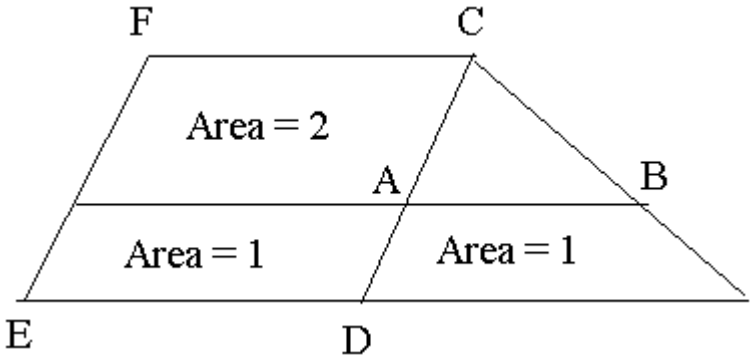
(a) Find all possible values for n.

(b) If 15 is replaced by an arbitrary positive integer x, show there is at least one value of n so that the $n^2 + 2n + 29$ pieces may be equally divided among x girls and n boys.

(c) In (b) give a condition on x in order that there is a unique value of n so that the $n^2 + 2n + 29$ pieces of candy may be divided equally among x girls and n boys.

(d) Find all values of x, $2 \le x \le 10$ for which there is a unique value for n in (b).

3. Given the figure below, if $CDEF$ is a parallelogram find the area of triangle ABC.



4. (a) Prove that for any positive integer n: $2^{2n+1} + 3^{2n+1}$ is divisible by 5

(b) Find positive integers a,b,c,d such that for any positive integer n: $2^{an+b} + 3^{cn+d}$ is divisible by 7.

5. Fischer and Spassky play a series of chess games. Fischer uses white pieces and Spassky uses black pieces the first game; they alternate colors thereafter with Fischer using white on the odd numbered games and black on the even numbered games. In each game the probability of a draw (tie) is 1/2. When Fischer uses white pieces the probability he wins is *r* and when he uses black pieces the probability he wins is *s*. Express in terms of *r* and *s* the following probabilities:

(a) The probability the first winning game is by Fischer using the

(a1) white pieces (a2) black pieces.

(b) The probability the first winning game is by Spassky.

(c) The probability the first two winning games are both by Fischer.

(d) The probability the first three winning games are all by Fischer with exactly two of the wins using white pieces.

(e) The probability the first ten winning games are all by Fischer using the black pieces for the third, sixth, and seventh wins and white pieces for the other wins.

□