David Essner Exam 3   1983-1984

1. Three die are rolled, the numbers 1,2,3,4,5,6 being equally likely to come up on each of the die. What is the probability that all three numbers are different?

   (a) 1/3   (b) 3/16   (c) 7/36   (d) ½   (e) 5/9

2. If x is a negative number, but the absolute value $|x|$ is very large, then

   $\dfrac{x-1/x+2}{3+x}$ is

   (a) very large   (b) near –1   (c) near 0   (d) near 1   (e) near 2/3

3.How many positive integers, less than 100, are divisible by both 4 and 6?

   (a) 4   (b) 6   (c) 8   (d) 10   (e) 12

4. If the fourth term of a geometric progression is a and the sixth term is b, then the first term is

   (a) a^4/b^6 (b) b^4/a^6 (c) $a^{5/2}/b^{3/2}$ (d) $a^{3/2}/b^{5/2}$ (e) $6a-4b$

5. For which numbers m do the equations $mx-y+1=0$ and $4x^2+y^2-4=0$ have at least one simultaneous real solution?

   (a) all   (b) none   (c) $m \le 2$ (d) $-2 \le m \le 2$ (e) $0 \le m \le 1$

6. Which of the following numbers is nearest 1?

   (a) $(1.01)^{10}$ (b) $(1.0001)^{100}$ (c) 1.101   (d) $(.99)^{100}$ (e) $(.9)^{10}$

7. A student took four one hour exams and a two hour final exam equivalent to two one hour exams. If the student averaged 80 on the four one hour exams and 84 on all exams, what was the score on the final exam?

   (a) 86   (b) 88   (c) 90   (d) 92   (e) 94

8. The altitude and one side of an equilateral triangle differ by 1. What is a possible value for the side?

   (a) $2\sqrt{3}+1$ (b) $3+\sqrt{2}$ (c) $4-2\sqrt{3}$ (d) $4\sqrt{3}+1$ (e) $\sqrt{3}/2$

9. Let N be a 4 digit number $abcd$ and M the 4 digit number $dcba$ obtained by reversing the digits of N (base 10 assumed). Then $M-N$ is an even number only if

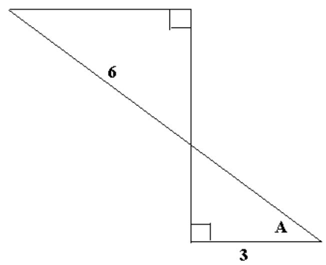
   (a) $d-a$ is even   (b) $c-b$ is odd   (c) $ad=bc$ (d) $a+d=b+c$

   (e) $a+b+c+d=0$

10. In the figure below, for what values of A does the lower triangle have greater area than the upper triangle?

(a) $A > 60°$ (b) $A > 45°$ (c) $A > 30°$ (d) none

(e) cannot determine from the given information.



11. What is the rate of investment such that the value of the investment, compounded n times per year, will double in one year?

(a) $2/(n + 1)$ (b) $\log_2 2n$ (c) $\log_n(2 - n)$ (d) $2^{1/n} - 1$ (e) $2 - 2^{1/n}$

12. Let A,B,C be statements which are either true or false. Given the hypotheses:

(i) if A then B and (ii) if C then B

then a valid conclusion is

(a) if C then A (b) if B then (A or C) (c) if not B then (A and C)

(d) if (A or B) then C (e) if not B then not (A or C)

13. A "full house" poker hand consists of a triple and a pair e.g. three jacks and two eights How many full house poker hands are possible in a standard deck of 52 cards?

(a) 64 (c) 3744 (c) 5280 (d) 6536 (e) 27,264

14. A triangle has two sides of length 3 and the altitude to the other side has length 2. The tangent of the angle between the sides of length 3 is

(a) $-4\sqrt{5}$ (b) $-\sqrt{5}$ (c) 2/3 (d) $\sqrt{5}/2$ (e) $6\sqrt{5}$

15. Given that the polynomial $x^4 + x^3 - 2x^2 + 4x - 24$ has 2 and $2i$ (where $i^2 = -1$) as two of its roots, then another root is

(a) $2 + 2i$ (b) 4 (c) -3 (d) -1 (e) $1 + i$

16. If the decimal number 1553 is expressed to the base 2, then there are how many 1's in the result?

(a) none (b) 2 (c) 4 (d) 7 (e) 11

17. Given three piles of coconuts, if 1/5 of the total number of coconuts is in the first pile, several sevenths are in the second pile, and 12 coconuts are in the third pile, what is the total number of coconuts?

(a) 105 (b) 120 (c) 140 (d) 420 (e) 700

18. How many positive integers less than 1000 include the digit 1 at least once?

(a) 142 (b) 271 (c) 300 (d) 410 (e) 80

19. The length of the common chord of two intersecting circles is 16. If the radii are 10 and 17, a possible value for the distance between the centers of the circles is

(a) 21 (b) 24 (c) 16 (d) 49/3 (e) 39/2

20. If $f(x) = 3^{2x+7}$ then $f(x + 1) – f(x) =$

(a) 3^9 (b) $8f(x)$ (c) $f(2x) + 7$ (d) $9f(x)$ (e) 3

21. To 50 ounces of a solution of equal parts water and acid, x ounces of water are added to yield a solution of 40 % acid. The value of x is

(a) 20 (b) 15/2 (c) 25/2 (d) 10 (e) 15

22. The number of solution pairs of positive integers of the equation $5x + 7y = 465$ is

(a) none (b) 9 (c) 13 (d) 43 (e) 66

23. Joe pays \$$x$ for an article and sells it to Tom for a 5% profit. Tom then sells it to Bill at a 10% loss. If the difference between the amounts Tom and Bill paid is 70 cents, the value of x is

(a) 14 (b) 7/3 (c) 5/7 (d) 30/7 (e) 20/3

24. Let $f(x) = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$; then $f(1/3) =$

(a) $5^3/3^5$ (b) $(4/3)^5$ $(1 + (2/3)^5$ (d) $3 – (1/3)^5$ (e) $(5/3)^4$

25. ·What is the number base in which the subtraction 1000 – 440 = 40 is valid?

(a) 5 (b) 6 (c) 7 (d) 8 (e) 9

26. If $\log_{10} 2 = a$ and $\log_{10} 3 = b$ then $\log_5 12 =$

(a) $\frac{a+b}{1+a}$ (b) $\frac{2a+b}{1+a}$ (c) $\frac{a+2b}{1+a}$ (d) $\frac{2a+b}{1-a}$ (e) $\frac{a+2b}{1-a}$

27. The plane region which is determined by the three inequalities $y < 2x$, $y > x$, $y < 6 – x$ contains how many points (m,n) where m and n are both integers?

(a) none (b) 1 (c) 2 (d) 5 (e) infinitely many

28. A square S_1 is inscribed in a circle C of radius 1 and a second square S_2 is circumscribed about C. The average of the areas of S_1 and S_2 is

(a) 3 (b) $2 + \sqrt{2}$ (c) $6 - \pi$ (d) $\sqrt{2} + \sqrt{3}$ (e) $(\pi + 3)/2$

29. Tom starts with the amount 64 cents and makes 5 bets. On each bet he either wins or loses half of his amount. If he wins 2 of the 5 bets and loses the other 3, then Tom ends with (in cents)

(a) 18 (b) 27 (c) 36 (d) 48 (e) depends on the order in which he wins or loses

30. In a race Jack ran 4/3 as fast as John, and finished in 1/9 hour less time. If Jack and John ran the same distance, how many minutes did it take Jack to run the race?

(a) 15 (b) 20 (c) 27 (d) 36 (e) 42

31. Let f_1 be the fractional linear transformation $f_1(x) = \frac{2x-1}{x+1}$. Define $f_{n+1}(x) = f_1(f_n(x))$ for $n = 1,2,\ldots$. Given that $f_{35} = f_5$, what is $f_{28}(x)$?

(a) x (b) $1/x$ (c) $\frac{x-1}{x}$ (d) $\frac{1}{1-x}$ (e) none of (a)-(d)

32. The minimum value of the quotient of a (base ten) number consisting of 3 non-zero digits divided by the sum of its digits is

(a) 10 (b) 19/2 (c) 10 9/19 (d) 21/2 (e) 10 9/11

33. When the number 2^{1000} is divided by 13, the remainder in the division is

(a) 1 (b) 2 (c) 3 (d) 7 (e) 11

34. The roots of $64x^3 - 144x^2 + 92x - 15 = 0$ are in arithmetic progresion. The difference between the largest and smallest roots is

(a) 2 (b) 1 (c) 4 (d) 3/8 (e) ¼

35. Let n be the number of integer values of x such that

$$P = x^4 + 6x^3 + 11x^2 + 3x + 31$$

is the square of an integer. Then n =

(a) 4 (b) 3 (c) 2 (d) 1 (e) 0