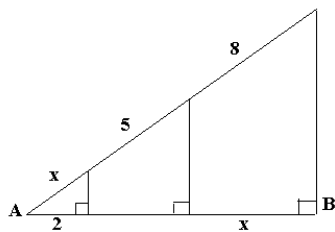


David Essner Exam 4 1984-1985

- Four coins are tossed. What is the probability that at least two heads occur?
 (a) $1/2$ (b) $11/16$ (c) $3/8$ (d) $1/3$ (e) $5/8$
- If the sum and difference of the first two terms of a geometric progression are respectively 18 and 12, then the third term of the progression is
 (a) 24 (b) 36 (c) 45 (d) 64 (e) 75
- Let $A(n)$ be the statement $n^2 < 49$ and $B(n)$ be the statement $2n^2 + n - 3 > 0$. Then the sum of all positive integers n for which $A(n)$ is true and $B(n)$ is false is
 (a) 15 (b) 6 (c) 10 (d) 1 (e) 20
- If x is positive and very large then $\frac{3 - x + 5x^2}{6 + 7x - 8x^3}$ is
 (a) near $1/2$ (b) near $-5/8$ (c) negative and near 0 (d) positive and near 0
 (e) very large
- If Jane scores 82 on exam 1 and 90 on exam 2, then what must be her average on the next 3 exams to have an average of 92 for the 5 exams?
 (a) 95 (b) 94 (c) 96 (d) 98 (e) 100
- An urn contains 10 balls numbered 1 through 10. A ball is drawn at random from the urn, placed back in the urn, and then a ball is again drawn at random from the urn. What is the probability that the number on the last ball drawn is less than the number on the first ball drawn?
 (a) $5/11$ (b) $9/20$ (c) $10/21$ (d) $1/2$ (e) $11/20$
- The student-faculty ratio at a certain school is 30 to 1. If the faculty were increased by 20% then the resulting ratio would be
 (a) 25 to 1 (b) 24 to 1 (c) 28 to 1 (d) 36 to 1
 (e) depends on the original number of students and faculty
- The length of the segment AB in the figure below is
 (a) 9 (b) $15/2$ (c) $17/2$ (d) $8 + \sqrt{2}$ (e) $15 - \sqrt{2}$



9. A man makes a trip by auto at an average speed of 40 mph. In order that he have an average speed of 50 mph for the entire trip, what speed in mph must be his average speed for the return trip over the same route?
 (a) 55 (b) 60 (c) $66\frac{2}{3}$ (d) $62\frac{1}{2}$ (e) depends on the trip distance
10. Which of the following is equivalent to the statement "If not A then (B or not C)" ?
 (a) If (not B and C) then not A (b) If A then (not B or C)
 (c) If (not A and B) then not C (d) If (not B and C) then A
 (e) If (B and not C) then A
11. Which of the following numbers is the largest?
 (a) 1.1 (b) $(1.01)^{10}$ (c) $(1.001)^{100}$ (d) $1/9$ (e) $(1/.99)^{10}$
12. How many positive integers less than 100 have at least three different prime divisors?
 (a) 4 (b) 8 (c) 6 (d) 5 (e) 10
13. Among all points (x,y) where x and y simultaneously satisfy the two inequalities
 $(i) x^2 + 2x + y^2 \leq 4$ and $(ii) y \leq |x|$
 the value of y which is the largest is
 (a) 2 (b) $\sqrt{3}/2$ (c) $1 + \sqrt{5}$ (d) $\sqrt{2}$ (e) 3
14. Beginning with 100 ounces of 50% salt solution, 50 ounces of solution is removed and 50 ounces of plain water is then added. This is then repeated for a total of ten times. The resulting solution is approximately what percent salt?
 (a) 5 (b) 1 (c) 50 (d) .05 (e) .01
15. In a race Joe went twice as far as Bill, and at a rate of 10 feet per minute faster than Bill. If Bill took 6 minutes and Joe took 5 minutes for the race, what was the rate of Bill (to the nearest integer) in feet per minute?
 (a) 3 (b) 5 (c) 7 (d) 8 (e) 9
16. Let a_n be a sequence such that $a_1 = 1$, $a_4 = 13$ and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. Then $a_6 =$
 (a) 18 (b) 26 (c) 33 (d) many possible answers (e) no possible answers
17. Jack pays $\$P$, plus $r\%$ tax, for an article; he then sells the article to Bill at a loss of $s\%$ (loss based on Jack's total cost). If $\$Q$ is the amount Bill paid then $100(1 - Q/P) =$
 (a) $rs/100 + (s - r)$ (b) $(r + s - rs)/100$ (c) $(r - s + rs)/100$ (d) $rs + (s - r)/100$
 (e) $r + s - rs/100$
18. What investment at an annual rate of 12% compounded 4 times a year will be worth \$1,000 in 10 years?
 (a) $1,000(1.03)^{-40}$ (b) $4,000(1.12)^{-10}$ (c) $100(1.03)^4$
 (d) $10,000(1.03)^{-12}$ (e) $1,000(1.40)^{-12}$

19. Given that a and x are positive numbers which simultaneously satisfy the equations $2^{\log_a x} = 8$ and $\log_6 x = \log_6 a + 1$ then $a =$

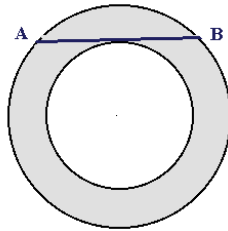
- (a) $2\sqrt{2}$ (b) $\sqrt{6}$ (c) $(\sqrt{2} + 1)/6$ (d) $(\sqrt{2} - 1)/6$ (e) 32

20. An isosceles triangle has two sides of length 10 with 45° as the angle between the sides. The third side has length

- (a) $5\sqrt{2}$ (b) $5(2 - \sqrt{2})$ (c) $5(\sqrt{3} - 1)$ (d) $5 + \sqrt{\sqrt{2} + 2}$
 (e) $10\sqrt{2 - \sqrt{2}}$

21. The distance from A to B (in the figure below) is 2; the area of the shaded region is

- (a) π (b) 2π (c) 3π (d) $3\pi/2$ (e) 4π



22. The system of equations $x + y + z = 1$, $x - y + 2z = 3$, $2x + 4y + z = t$

- (a) has exactly one solution for each value of t
 (b) has infinitely many solutions for each value of t
 (c) has no solution for any value of t
 (d) has infinitely many solutions for exactly one value of t
 (e) has exactly one solution for exactly one value of t

23. For N a positive integer let $T(N) = N/2$ if N is even and $T(N) = (N - 1)/2$ if N is odd. For how many positive integers is $T^3(N) = 17$ where $T^3(N) = T(T(T(N)))$?

- (a) 1 (b) 2 (c) 4 (d) 8 (e) 16

24. Given the cubic equation $x^3 + Ax^2 + Bx + C = 0$, if S is the sum of the roots and P is the product of the roots then

- (a) $C = PS$ (b) $B = P + S$ (c) $A = P/S$ (d) $AB = PS$ (e) $A + C = -P - S$

25. If the integer N is initially assigned the value 1, and is then three successive times replaced by the square of one more than its value, then the resulting number is

- (a) 18 (b) 36 (c) 64 (d) 128 (e) 676

26. How many positive integers less than 1000 have the sum of its digits equal to 10?

- (a) 47 (b) 55 (c) 63 (d) 72 (e) 81

27. For positive integers A and B let $A \bmod B$ denote the remainder when A is divided by B (e.g. $23 \bmod 4 = 3$). Thus if $N = abc$ is a 3 digit number, where a, b, c are the digits to base 10, then $c =$

- (a) $(N \bmod 100) \bmod 10$ (b) $N \bmod 1000 - N \bmod 100 + N \bmod 10$
 (c) $N \bmod 100 + N \bmod 10$ (d) $\frac{N}{N \bmod 100} \div N \bmod 10$
 (e) $(N \bmod 100)(N \bmod 10)$

28. If $x^{70} + x^{10} + x - 5$ is divided by $x^2 - 1$ the remainder is

- (a) $x - 3$ (b) $2x + 7$ (c) $5x - 1$ (d) $x + 2$ (e) $5 - 2x$

29. The infinite sequence 11, 111, 1111, 11111, ... includes how many perfect squares?

- (a) 1 (b) 0 (c) 2 (d) more than two but a finite number (e) infinitely many

30. The units digit of 27 is 7, of 356 is 6. What is the units digit of the number 3^{1000} ?

- (a) 1 (b) 3 (c) 5 (d) 7 (e) 9

31. For N a positive integer let $S_N = 2 + 4 + 6 + \dots + 2N$. For how many integers N is S_N a perfect square?

- (a) 1 (b) 0 (c) 2 (d) more than two but a finite number (e) infinitely many

32. For positive integers M, N , where $M \geq N$, let G be the greatest common divisor of M and N . Then among the number pairs

- (I) N and $M + 2N$ (II) MN and $M + N$ (III) N and $M - N$
 (IV) $M + N$ and $M - N$

the number G is also the greatest common divisor of

- (a) I and III only (b) I, II, IV only (c) III and IV only (d) III only
 (e) none of the pairs