David Essner Exam 5 1985-1986

1. Given a set of 5 points in the plane, no three of which are collinear, how many different triangles can be formed using the 5 points?
   (a) 5 (b) 8 (c) 10 (d) 15 (e) 125

2. A coin is tossed until it comes up heads. What is the probability that 4 or more tosses are required?
   (a) 1/8 (b) 3/16 (c) ¼ (d) 3/64 (e) 5/64

3. If $x = 1$ and $x = -1$ are solutions of $f(x) = 0$, where $f(x)$ is a polynomial, then a divisor of $f(x)$ is
   (a) x (b) $x^2 + 1$ (c) $2x + 1$ (d) $x + 2$ (e) $x^2 - 1$

4. If the sum of the first four terms of an arithmetic progression is 32, and the sum of the first 6 terms is 60, then the first term is
   (a) 3 (b) 5 (c) -17/3 (d) 19/2 (e) –10

5. If x is a negative number near 0 then $\dfrac{3/x + 6/x^2 - 20/x^3}{1/x - 3/x^2 + 5/x^3}$ is
   (a) negative and large in magnitude (b) near 0 (c) near –3 (d) near 2
   (e) near –4

6. A class consists of 21 boys and 9 girls. On an exam the average of the class was 84 and the average of the boys was 80. The integer nearest to the average of the girls is
   (a) 86 (b) 87 (c) 89 (d) 91 (e) 93

7. The graph of $y = f(x)$ may be obtained from the graph of $y = f(2(x))$ by
   (a) shifting it upwards (b) shifting it to the right (c) stretching it horizontally
   (d) reflecting it about the line $y = 2x$ (e) a rotation

8. The graph of which equation is symmetric about the x axis?
   (a) $x^2 + y = 0$ (b) $x^2 - y^3 = 0$ (c) $x^2 + y^2 - \sin y = 0$ (d) $x^2 \cos y + y = 0$
   (e) $x^2 + y \sin y = 0$

9. If $cd \neq 0$ then the graph of $x^2 + y^2 + cx + d = 0$ is a circle
   (a) if $c < 2d$ (b) if $c^2 > 4d$ (c) if $c^2 > d + 4$ (d) always (e) never

10. To 100 ounces of a solution of equal parts of water and acid, x ounces of acid are added to yield a solution of 80% acid. Then y ounces of water are added to bring the solution to 40 % acid. A formula relating x and y is
   (a) $y = 80 + .4x$ (b) $y = 25 + 3x/2$ (c) $y = .4(100 + 1.8x)$
   (d) $y = .8(100 + 1.4x)$ (e) $y = 140 + .8x$

11. Which of the numbers is nearest $(1.05)^{10} - 1.5$?
   (a) 1/920 (b) 1/75 (c) 1/54 (d) 1/9 (e) ½

12. Bill has \$3 and Tom has \$1. They toss a coin, and the loser pays the winner \$1. This continues until either Bill or Tom has lost all his money. What is the probability there is at most 4 coin tosses before the betting is ended?

(a) 63/64 (b) 15/16 (c) 7/8 (d) ¾ (e) ½

13. The amount which must be invested at an annual rate of 12% compounded 4 times a year to have value 1,000 in 10 years is 1,000/Q where Q =

(a) $(1.12)^{30}$ (b) 1.48 (c) 30 $\log_{12} .03$ (d) $(1.03)^{40}$ (e) 12 $\log_{10} 40$

14. The statement 'If P then (Q and R)' is true provided

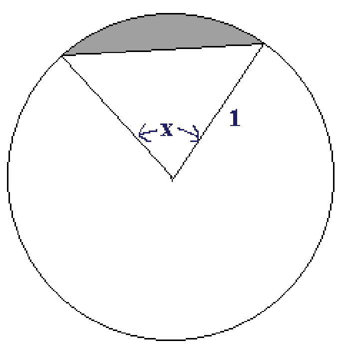
(a) P is false and Q is false (b) P is true and Q is true (c) P is true or R is false (d) (P or Q) is true or R is true (e) (P and Q) is false and R is true

15. Bill and Joe run a race of 1,000 feet at constant speeds. Bill starts 20 seconds ahead of Joe, and Joe passes Bill after 800 feet. If Bill ran the race in 2 minutes, how many minutes did it take Joe to run the race?

(a) 7/4 (b) 8/5 (c) 27/16 (d) 19/12 (e) 3/2

16. The area of the shaded portion of the figure below is (x is in radians)

(a) $(2 \cos x - 1)/2$ (b) $(\sin x + 1)/2$ (c) $(\tan x - 1)/2$ (d) $\sin x + \cos x$
(e) $(x - \sin x)/2$



17. If $N > 1{,}000$ then which number is the largest?

(a) N^{100} (b) 2^N (c) $\log_2 N$ (d) $N^{30} + N^{40} + N^{50}$ (e) $N^{1/N}$

18. A set of 8 elements has how many different subsets with 6 or more elements?

(a) 37 (b) 52 (c) 60 (d) 84 (e) 224

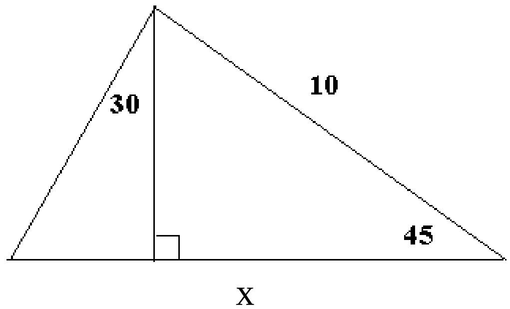
19. How many positive integers divide both the integers 360 and 600?

(a) 5 (b) 11 (c) 16 (d) 21 (e) 24

20. In the figure below x =

(a) $20/(1+\sqrt{3})$ (b) $5(\sqrt{2}+2\sqrt{3})$ (c) $5\sqrt{2}(1+1/\sqrt{3})$

(d) $10(1+\sqrt{3})/\sqrt{2}$ (e) $10\sqrt{3}(1+\sqrt{2})$



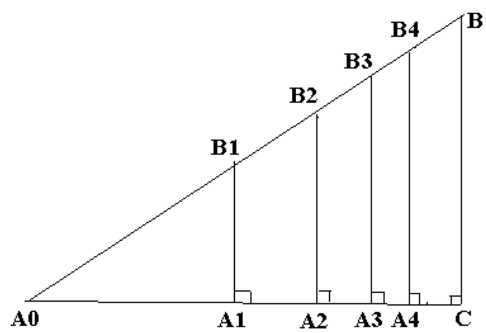
21. If the two digit numbers '2x' and 'y1' are added, then the result is greater than 100 provided

(a) $x + y \geq 15$ (b) $10y + x \geq 80$ (c) $10x + 7y \geq 120$

(d) $x \geq 8$ and $y \geq 6$ (e) $x + 7y > 50$

22. In the figure below $BC = A_0C = 32$ and $A_NC = 2A_{N+1}C$ for $N = 0,1,2,3,\ldots$.
What is the area of the quadrilateral $A_3B_3A_4B_4$?

(a) 11 (b) 39/2 (c) 27 (d0 99/2 (e) 58



23. In the Cartesian plane let T be the triangle with vertices (0,0), (1,0) and (1,3). How many points (x,y) are there inside T (do not include points on the sides of T) where both x and y may each be written in the form P/Q, $Q < 5$, where P and Q are positive integers?

(a) 26 (b) 40 (c) 50 (d) 62 (e) 81

24. If $\log_2(\log_2(\log_3 x)) = \log_2(\log_3(\log_2(y))) = \log_3(\log_2(\log_2 z)) = 1$ then $x + y + z$ =

(a) 36 (b) 64 (c) 324 (d) 656 (e) 849

25. A solution of the inequality $|x - 3| + |x + 2| \leq 6$ is

(a) $-5/2 \leq x \leq 7/2$ (b) $x \geq 6$ or $x \leq 1$ (c) $x \geq 3$ or $x \leq 2$

(d) $2 \leq x \leq 3$ (e) $x \geq 3/2$ or $x \leq -1/2$

26. Give the smallest number among (a)-(e) which makes true the statement:

‘In order to determine if the number 211 is a prime number it is sufficient to show 211 is not divisible by each prime number up to and including ‘

(a) 11 (b) 13 (c) 29 (d) 107 (e) 209

27. Given the recurrence relation $f(0) = 2$, $f(1) = 1$, $f(n) = f(n-1) - f(n-2)$ for $n > 2$, then $f(100) =$

(a) –101 (b) –2 (c) 2 (d) 1 (e) –1

28. Given a pile of x coconuts, if the pile is subdivided into 3 equal piles then there is 1 coconut left over, if it is subdivided into 5 equal piles then there are 2 coconuts left over, and if it divided into 7 equal piles then there are 3 coconuts left over. If $x < 100$ then the sum of the digits of the integer x is

(a) 3 (b) 4 (c) 6 (d) 7 (e) 9

29. The value of the sum $\sum_{n=1}^{100} \frac{1}{n(n+1)}$ is

(a) 17/4 (b) 23/17 (c) 1325/97 (d) 100/101 (e) 199/99

30. A student writes the number 1 on a chalkboard. Then in succession 10 students erase the number on the board and replace it by 1 less than three times the number on the board. The resulting number is

(a) $3^{10} - 10$ (b) 2^{10} (c) $(10^3 - 1)/2$ (d) $(3^{10} + 1)/2$ (e) $3(10^2 - 1)$

31. On the set of positive real numbers let the transformation T be defined by T(x) = 2/x. Also let $T^{n+1} = T(T^n(x))$, $n = 1,2,3,\ldots$ where $T^1 = T$. Then $T^{10}(x) =$

(a) $(2/x)^{10}$ (b) $2/x^{10}$ (c) $2^{10}/x$ (d) $(x/2)^{10}$ (e) x

32. The smallest value of x for which $1{,}400\,x = N^3$ for some integer N is

(a) 42 (b) 85 (c) 212 (d) 245 (e) 536

33. How many integers between 100 and 999 have their digits in increasing order (i.e. if the integer is xyz then $x < y < z$)?

(a) 78 (b) 84 (c) 106 (d) 121 (e) 186

34. If m and n are both perfect squares and $m - n = 29$ then $m + n =$

(a) 117 (b) 171 (c) 366 (d) 421 (e) 1048