David Essner Exam 6   1986-1987

1. A student took a sequence of exams, and the scores on the exams formed an arithmetic progression. If the average of the first two exams was 78, and the average of the first 3 exams was 80 then the difference in the scores of the first and second exams was

(a) 2   (b) 3   (c) 4   (d) 6   (e) cannot be determined from the given information

2. If r is a negative number which is very close to 0 then the ratio $\frac{1-r-r^2}{5r}$ is

(a) negative and large in magnitude   (b) positive and near 0
(c) negative and near 0   (d) near 1/5   (e) near -1/5

3. The system of equations

$$\begin{aligned} x + y - z &= 8; \\ 2x + 3y + z &= 9; \\ 2x + y - 5z &= t \end{aligned}$$

has a solution for x,y,z only if t =

(a) 0   (b) -7   (c) 13   (d) 23   (e) -12

4. The coefficient of x^6 in the expansion of $(1 + x + x^2 + x^3 + ... + x^{10})^3$ is

(a) 12   (b) 18   (c) 24   (d) 28   (e) 36

5. A bottle contains x ounces of water. The addition of y ounces of acid then produces a 30% solution, and the further addition of z ounces of acid produces a 60% solution. The ratio z/y =

(a) 1/2   (b) 2/3   (c) 3/2   (d) 2   (e) 5/2

6. If the graph of $y = f(x)$ contains the point (3,7) then the graph of $y = 3f(x - 2) + 5$ must contain the point

(a) (5,26)   (b) (9,14)   (c) (21,15)   (d) (20,7)   (e) 8,12)

7. The line $y = x + B$ is tangent to the ellipse $x^2 + 2x + y^2 = 0$ if B =

(a) $1 + \sqrt{2}$ (b) $2 - \sqrt{2}$ (c) $\sqrt{5} - 1$ (d) $-\sqrt{2}$ (e) $\sqrt{5} - \sqrt{2}$

8. Which of the following is the best approximation for $\sqrt{1.09}$?

(a) 1.03   (b) 1.045   (c) 1.3   (d) 1.003   (e) 1.081

9. An urn contains a red ball, a green ball and a blue ball. Three times a ball is drawn at random from the urn and then returned to the urn. What is the probability each of the three balls was drawn.?

(a) 1/2   (b) 1/3   (c) 1/9   (d) 2/9   (e) 7/27

10. If for positive integers M and N , M mod N is the remainder from the division of M by N then 46 mod ((60 mod 31) mod 11) equals

(a) 13   (b) 9   (c) 8   (d) 4   (e) 0

11. A person drives from town *A* to town *B* at 30 mph, then from *B* to *A* at 40 mph, and again from *A* to *B* at 50 mph. What is the average speed, in mph of the three trips?
(a) 40 (b) 1800/47 (c) 79/2 (d) 2360/61 (e) depends on the distance from *A* to B

12. Tom bought some pens and paid a 5% sales tax. If there were no tax then he could have bought 3 more pens for the same amount of money. How many pens did he buy?
(a) 30 (b) 33 (c) 45 (d) 20 (e) 60

13. Given the triangle in the Cartesian plane with vertices (0,0), (0,10) and (20,0), how many points (*m*,*n*) are interior to the triangle if *m* and *n* are both integers (do not include points on the perimeter of the triangle)?
(a) 72 (b) 81 (c) 92 (d) 100 (e) 108

14. At what real number interest rate *r*, compounded two times per year, will an investment triple in 10 years?
(a) 1/8 (b) $2^{3/20} - 1$ (c) $2(3^{1/20} - 1)$ (d) $\log_{10}(1 + 3/2^{10})$ (e) $10\, e^{2/3}$

15. Given the five numbers $(2^4)^8$; $8^{(2^4)}$; $(8^4)^2$; $4^{(8^2)}$; $8^{(4^2)}$
the ratio of the largest to the smallest is
(a) 4^{52} (b) 2^{81} (c) 8^{39} (d) 16^{23} (e) 2^{1020}

16. If the quadratic equation $x^2 + mx + n = 0$ has integer roots, and *m* and *n* are integers, then which of the following is not possible?
(a) $mn > 0$ (b) *n* is odd and *m* is even (c) *n* is even and *m* is odd
(d) *n* is odd and *m* is odd (e) *n* is even and *m* is even

17. Bill plays roulette 10 times. He bets $1 the first time and thereafter he bets $1 if he won the previous bet and he bets double the previous bet if he lost the previous bet. Of the ten bets Bill wins 2 times and loses 8 times. The best he could possibly have done is
(a) lose $32 (b) lose $12 (c) lose $6 (d) break even (e) win $2

18. Which is a valid conclusion to the argument whose hypotheses are:
(*I*) If *P* then *Q* (*II*) *Q* and not *P* ?
(a) *P* or (not *Q*) (b) *P* and *Q* (c) If *Q* then *P* (d) *P* or *Q* (e) *P* and (not *Q*)

19. Bill and Tom run a race. Bill starts 100 feet behind Tom, runs 10/9 as fast as Tom, and they finish in a tie. How many feet did Bill run?
(a) 1000 (b) 1100 (c) 810 (d) 911
(e) cannot tell from the given information

20. A triangle has angles of 75°, 60°, and 45° and the area is 10. If the length of the side opposite the 60° angle is 6 then the length of the side opposite the 75° angle is
(a) $\frac{6}{\sqrt{2}}$ (b) $\frac{20}{\sqrt{3}+1}$ (c) $\frac{10+\sqrt{3}}{3\sqrt{2}}$ (d) $3\sqrt{2}\,(1+ 1/\sqrt{3})$ (e) $\frac{8\sqrt{3}}{5\sqrt{2}}$

21 If $S = a - b + c - d$, where $a > b > c > d > 0$, then which of the statements

(I) $s < a$ (II) $s > d$ (III) $s > 0$

are necessarily true?

(a) *III* only (b) *I* and *II* only (c) *I* and *III* only (d) all (e) none

22. Given that $\log_3 8 = A$ and $\log_{16} 5 = B$ then $\log_2 15 =$

(a) $\frac{3+4AB}{A}$ (b) 5A + B/3 (c) $\frac{A+3B}{5}$ (d) $\frac{4A+5B}{3}$ (e) $\frac{A+5B}{4}$

23. Given the difference equation $F(n) = F(n-1) - F(n-2)$, if $F(1) = F(2) = 1$ then $F(1000) =$

(a) -999 (b) -1 (c) 0 (d) 1 (e) 500

24. How many 8 letter words, consisting only of the letters *a* and *b*, are there which do not have two consecutive *a*'s?

(a) 13 (b) 21 (c) 36 (d) 42 (e) 55

25. The area of the set of points (x,y) such that $|y - x| \leq 3$ and $|y + x| \leq 2$ is

(a) 6 (b) 9 (c) 12 (d) 15 (e) 18

26. The equation $x^2 + x + 1 = A$ has exactly one real root provided $A =$

(a) 0 (b) 1 (c) -1 (d) 2/3 (e) 3/4

27. Given the equation $19m + 87n = 1987$, where m and n are positive integers, one solution is $m = 100$, $n = 1$. There is exactly one other solution for m and n, and in this case $m + n =$

(a) 25 (b) 30 (c) 33 (d) 37 (e) 39

28. The number 20! (= 1x2x3x...x20) is best approximated by which of the following?

(a) $3(10)^{10}$ (b) $5(10)^{14}$ (c) $2(10)^{18}$ (d) $4(10)^{22}$ (e) $6(10)^{26}$

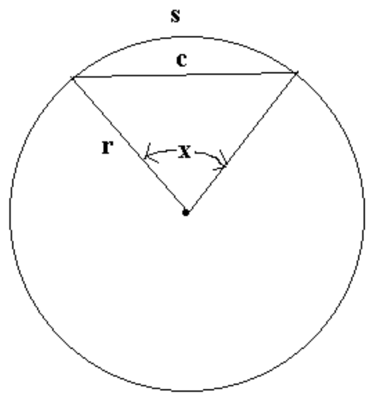
29. Let $S = \{1,2,3,...,21\}$ be the set of all integers from 1 to 21 inclusive. In how many ways can two different integers be selected from S so that the sum is even?

(a) 92 (b) 100 (c) 108 (d) 116 (e) 134

30. In the figure below the ratio s/c depends

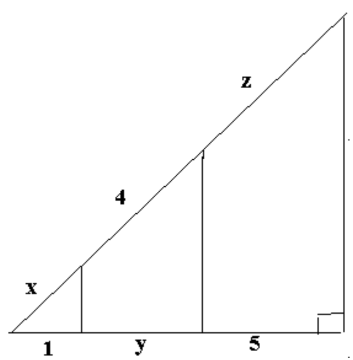
(a) on neither r nor x (b) on both of r and x (c) on r but not on x

(d) on x but not on r (e) none of (a),(b),(c),(d)



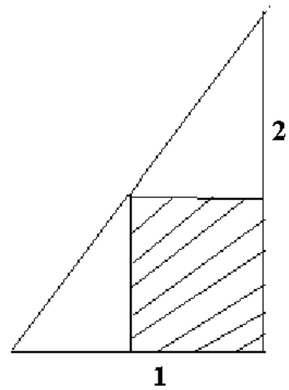
31. In the figure below if x,y,z are all integers then the largest possible value of the product xyz is

(a) 20 (b) 32 (c) 36 (d) 48 (e) 80



32. In the figure below the largest possible area of the shaded rectangle is

(a) 1/4 (b) 3/8 (c) 1/2 (d) 3/4 (e) 7/8



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