David Essner Exam 7 1987-1988

1. Given a set of four numbers, if one is 20, another is 40, the other two differ by 4 and the average of the four numbers is 35, then the largest number in the set is

(a) 40 (b) 41 (c) 42 (d) 43 (e) 44

2. If the lines $ax + by = c$ and $dx + ey = f$ are perpendicular then

(a) $ae = -1$ (b) $c = -1/f$ (c) $ad + be = 0$ (d) $af + dc = 0$ (e) $ab = -1/de$

3. If the system of equations $x + y + z = 6$
$x - y + z = 4$
$x + rz = 5$
has more than one solution for x,y,z then $r =$

(a) 5 (b) 4 (c) 1 (d) 0 (e) -2

4. Given that x is a small positive number and y is a large (in magnitude) negative number then $\frac{1/x + y - 5}{2/x - 4y + 2}$ is

(a) near 1/2 (b) near 1/4 (c) near -5/2 (d) near -1/4
(e) cannot tell from the given information

5. If $y = 10^8 + 1$ and $x = 10^3 - 1$ then $\frac{1}{1/x - 1/y}$ is near

(a) 10^{-5} (b) 10^{11} (c) 10^3 (d) $10^{3/8}$ (e) 10^5

6. If the graph of $x^2 + (y - 3)^2 = 10$ is shifted 4 units to the right and 5 units down then it contains the point

(a) (5,1) (b) 6,-1) (c) (2,0) (d) (6,-3) (e) (3,2)

7. John wishes to simulate the drawing of a card from a standard deck of 52 cards by repeatedly tossing a coin. What is the fewest number of times the coin must be tossed in order that different cards may be represented by different sequences of heads and tails?

(a) 6 (b) 9 (c) 12 (d) 18 (e) 26

8. Given that the implication "If P then Q" is false then which of the following implications must also be false?

(a) If Q then P (b) If Q then (not P) (c) If (P and Q) then Q
(d) If (not P) then (P or Q) (e) If (not Q) then (not (P or Q))

9. An amount of $1,000 is invested for one year at an annual rate of interest of 9%. At the end of the year 30% of the earned interest is withdrawn for payment of income tax. The value of the investment is then

(a) $1,060 (b) $1,027 (c) $1,600 (d) $1,063 (e) $1,021

10. Tom and Bill together run a relay for a total of 4 miles. Bill runs at 6 mph and Tom at 8 mph. If Tom runs for twice as much time as Bill, for how many hours did Bill run?

(a) 5/32 (b) 1/3 (c) 3/16 (d) 2/11 (e) 5/19

11. Given that B and C are integers from 0 to 9 inclusive and the product of the three digit integer $B2C$ and the two digit integer CB is 23,871 then $B + C =$

(a) 10 (b) 9 (c) 5 (d) 13 (e) 7

12. Given that the second term of a geometric series is 1/3 and the fourth term is 4/27 then the third term is

(a) 8/27 (b) 4/9 (c) 5/27 (d) 4/81 (e) 2/9

13. Five circles are drawn in the plane, dividing the plane into regions (two points are in the same region if they are the endpoints of a line segment which does not intersect any of the circles). What is the largest possible number of regions?

(a) 32 (b) 28 (c) 22 (d) 15 (e) 8

14. If $2^{(x^n)} = y$ then x is the following exponent of 2 (each log has base 2)

(a) $\frac{\log y - 1}{n}$ (b) $\frac{\log(\log y)}{n}$ (c) log (y/n) (d) $\frac{\log y}{\log n}$

(e) log(log(ny + 1))

15. If $x,y > 0$ and y/x is large then $\sqrt{y - x}$ is approximately

(a) $\sqrt{y} - \sqrt{x}$ (b) $y - \frac{x}{2\sqrt{y}}$ (c) $\frac{\sqrt{x}}{4y^2}$ (d) $\sqrt{y} - \frac{x}{2\sqrt{y}}$ (e) $\sqrt{y} - \sqrt{x/y}$

16. The line in the Cartesian plane through the point (a,b) and having slope 3 intersects the circle $x^2 + y^2 = 1$ in at least one point provided

(a) $|b - a| \le \sqrt{10}$ (b) $a^2 + b^2 \le 9$ (c) $|a - 3b| \ge 2$ (d) $|b| \le |9a + 1|$

(e) $|a^2 - b^2| \le 3\sqrt{2}$

17. How many integers between 1 and 10,000 inclusive have each of 2,3,5,7 as a single divisor? (e.g. 2 but not 2^2 divides such an integer)

(a) 12 (b) 19 (c) 26 (d) 36 (e) 47

18. If $0 < x < 1$ and $S = 1 + x + x^2 + x^3$ then which of the statements *I,II,III* must be true?

(*I*) $S > 1/x$ (*II*) $S < \frac{1}{1-x}$ (*III*) $S < \frac{7}{1+x}$

(a) *I,III* only (b) *II* only (c) *III* only (d) none (e) all three

19. Given the equation $4x + 3y = 529$ the number of solution pairs (x,y) where x and y are positive integers is

(a) 7 (b) 17 (c) 23 (d) 33 (e) 43

20. A sequence of integers is such that the sum of any two succesive integers is positive and the sum of any three successive integers is negative. The maximum value for n is

(a) 2 (b) 3 (c) 4 (d) 6 (e) larger than 6

21. For each integer $I = 1,2,3,...,10$ if I is odd then person $I + 1$ walks twice as far as person I, and if I is even then person $I + 1$ walks 1/3 as far as person I. If person 10 walks 1 mile then person 1 walks how many miles?

(a)3232323232 (b) $(1/2 + 3)^5$ (c) $(3/2)^{10}$ (d) $(2/3)^{10}$ (e) 81/32

22. A round robin tournment is one in which each team plays each other team exactly once. How many games are played in a round robin tournament of 8 baseball teams?

(a) 64 (b) 56 (c) 32 (d) 28 (e) 15

23. If $i = \sqrt{-1}$ then among all complex numbers $z = a + bi$ such that $z^3 = i$ the largest value of b is

(a) 1/3 (b) 1/4 (c) 2/3 (d) 1/2 (e) 3/2

24. An urn contains 3 black balls and 3 red balls. A ball is drawn from the urn; it is returned to the urn only if it is red. A second ball is drawn from the urn; it also is returned to the urn only if it is red. A third ball is drawn from the urn; the probability it is red is

(a) 1/2 (b) 2/3 (c) 63/100 (d) 113/180 (e) 121/200

25. If the polynomial $x^3 + bx^2 + cx + d$ has all real roots, the sum of the roots is 4, the product of the roots is -6, and one of the roots is 3 then c =

(a) -3 (b) 5 (c) -4 (d) 1 (e) cannot tell from the given information

26. A robot can walk in steps of 1,2, or 3 feet. In how many ways can a robot walk 6 feet?

(a) 11 (b) 16 (c) 24 (d) 30 (e) 36

27. Given the difference equation $f(n) = f(n - 1) - 2f(n - 2)$ then $f(n) =$

(a) $f(n - 3) - 4f(n - 1)$ (b) $-f(n - 2) - 2f(n - 3)$ (c) $f(n - 2) + 3f(n - 3)$
(d) $f(n - 1) - 5f(n - 3)$ (e) $f(n - 2) - 4f(n - 3)$

28. If M,N,D are integers, where $D > 1$, then $M \equiv N \bmod D$ provided $M - N$ is divisible by D. Given that $M \equiv N \bmod D$ and $P \equiv Q \bmod D$ then of

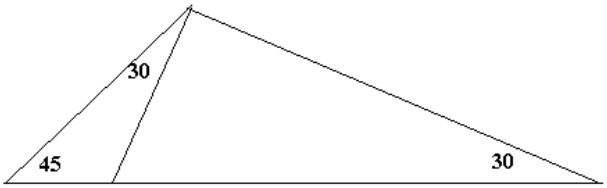
(I) $M + P \equiv (N + Q) \bmod D$ (II) $MP \equiv NQ \bmod D$
(III) $MQ \equiv NP \bmod D$ (IV) $MN \equiv PQ \bmod D$

which are not necessarily true?

(a) II,III only (b) I,IV only (c) III only (d) IV only (e) all must be true

29. The value of x in the Figure is

(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) 3/2 (d) $\sqrt{5} - 1$ (e) $\sqrt{3}/2$



30. In the Figure if $AF = 10$, $GH = 15$, $DF = 6$, $AB // DC$, $AD // EC // FG$, $AD \perp AB$, and $GH \perp BC$ (where // denotes 'is parallel to' and denotes $\perp$ 'is perpendicular to') then the area of $ABCD$ is

(a) 206 (b) 186 (c) 174 (d) 166 (e) 150