David Essner Exam 8 1988-1989

1. On an exam the 30 Honor Society students made an average of 80, and the 16 Math Society students made an average of 86. If 10 of the students who took the test were in both the Honor and Math Societies, and the average of all students was 81 then the average of those 10 in both Societies was

(a) 80 (b) 82 (c) 84 (d) 86 (e) 88

2. The sum of all solutions of the equation $x^2 - 7|x| + 3x = 0$ is

(a) -6 (b) -7/2 (c) 3 (d) 5 (e) 11

3. If S is the sum of all integers between 10 and 200 which are divisible by 7 then S =

(a) 2513 (b) 2772 (c) 2835 (d) 2905 (e) 2947

4. The largest integer N such that $N! < 1{,}000{,}000$ (where $N! = 1x2x3x...xN$) is

(a) 7 (b) 9 (c) 10 (d) 11 (e) 15

5. Given the mathematical operations * and # defined by

$a*b = a + 2b$ and $a\#b = a + b - 2$

which of the properties A,B,C,D are true?

A: * is commutative B: # is commutative C: * is associative D: # is associative

(a) A,B only (b) B,C only (c) B,D only (d) all (e) none

6. The third and seventh terms of a geometric progression are respectively 3 and 48. The first term is then

(a) 1/3 (b) 12 (c) 1 (d) 3/4 (e) 1/16

7. If n is a positive integer and $p = n(n + 1)(n + 2)$ then the largest integer which must divide p is

(a) 1 (b) 2 (c) 3 (d) 6 (e) 8

8. If x is a very small positive number then $\dfrac{5/x + 3}{x^2 + 7x - 2/x}$ is

(a) a very large positive number (b) near 0 (c) near -5/2
(d) a very large negative number (e) none of (a),(b),(c),(d)

9. Three dice are rolled. What is the probability that the three numbers that occur are all different?

(a) 25/36 (b) 119/216 (c) 7/18 (d) 2/3 (e) 5/9

10. If it is known that 'if $y \le 3$ then $x > z$' then which of the following must be true?

(a) if $x > z$ then $y \le 3$ (b) $y > 3$ and $x \le z$ (c) $y > 3$ or $x > z$
(d) $x + 3 \ge y + z$ (e) if $y > 3$ then $x \le z$

11. Given the system of equations: $2x + 3y - 12z = -1$
$3x - 2y - 5z = 18$
$x - 5y + 7z = 19$
if $x = a$, $y = b$, $z = c$ is a solution then
(a) $a = 3c + 4$ (b) $a = 6c - 9$ (c) $a = 2c + 7$ (d) $a = 5c + 6$ (e) $a = 4c + 7$

12. If 20 ounces of 20% solution of alcohol in water is mixed with 10 ounces of 30% solution of alcohol in water, and then x ounces of the resulting mixture is combined with 10 ounces of 40% alcohol in water, then the final result is a mixture having 30% alcohol. The value of x in ounces is
(a) 12 (b) 15 (c) 18 (d) 81/5 (e) 63/4

13. If an equilateral triangle is inscribed in a circle of radius 1 then its area is
(a) $\frac{3\sqrt{3}}{4}$ (b) $\frac{2\pi}{3}$ (c) $\frac{9\sqrt{3}}{4}$ (d) $\frac{2\sqrt{5}}{3}$ (e) $\frac{2+\sqrt{3}}{2}$

14. The coefficient of x^8 in the expansion of $(x^3 + x^2 + x + 1)^4$ is
(a) 10 (b) 31 (c) 38 (d) 60 (e) 120

15. Given that one root of the equation $2x^4 + 5x^3 - 10x^2 + 5x - 12 = 0$ is x = i (where $i^2 = -1$), then the sum of all real roots is
(a) 0 (b) 5 (c) -5 (d) -5/2 (e) 13/2

16. If $f(n) = \frac{1}{n} - \frac{1}{n+2}$ then $f(1) + f(2) + ... + f(9) =$
(a) 47/33 (b) 69/51 (c) 87/26 (d) 72/55 (e) 17/12

17. The cube of which number is closest to 8.0036?
(a) 2.0012 (b) 2.0003 (c) 2.004 (d) 2.0006 (e) 2.06

18. The line $y = 3x + 4$ is tangent to the circle $x^2 + y^2 = r^2$ if $r =$
(a) $3/\sqrt{2}$ (b) 3/4 (c) $\sqrt{14}/3$ (d) $\sqrt{10/3}$ (e) $\sqrt{8/5}$

19. If $\log_2 xy^2 = 4$, $\log_2(y/z) = 3$ and $\log_2 xz^3 = -4$ then $x + y + z =$
(a) 21/2 (b) 19/3 (c) 15.8 (d) 25/4 (e) is undefined

20. An urn has 2 black and 3 red balls. Bill draws a ball from the urn and then puts it back only if it is black; Mary then draws a ball from the urn. The probability Mary draws a red ball is
(a) 3/7 (b) 12/25 (c) 15/36 (d) 13/30 (e) 27/50

21. If $\cos 2x = 1/4$ then $\tan^2 x =$
(a) 5/8 (b) 3/5 (c) 9/16 (d) 2/3 (e) 25/36

22. If $f(0) = 1$ and $f(n) = 2\,f(n - 1)$ for n odd and $f(n) = \log_4 f(n - 1)$ for n even then $f(10) =$
(a) 0 (b) 1 (c) log 512 (d) 2 (e) is undefined

23. Bill and John make 5 successive bets. The first bet is \$1 and thereafter if Bill wins the next bet is double the previous bet and if John wins it is the same as the previous bet. If Bill wins 2 of the 5 bets then the best John and Bill respectively can do is
(a) win \$6, win \$3 (b) win \$7, lose \$1 (c) win \$5, win \$3
(d) win \$4, lose \$6 (e) win \$9, break even

24. Which product is closest to 1?
(a) 1.01x0.9999999 (b) 1.001x0.999999 (c) 1.0001x0.99999
(d) 1.00001x0.9999 (e) 1.000001x0.999

25. If $f(1) = 1$, $f(n) = f(n/2) + 1$ if n is even and $f(n) = 1/f(n - 1)$ if n is odd then $f(50) =$
(a) 9/7 (b) 99/75 (c) 39/15 (d) 427/5 (e) 1050/27

26. A triangle has sides of length 9, 21, and x where x is an integer. If all angles of the triangle are acute angles then the smallest possible value for x is
(a) 13 (b) 16 (c) 18 (d) 19 (e) 24

27. If 3 divides $n + 2$ then 3 must also divide which of the following?
(a) $n^2 + 6n - 7$ (b) $n^2 + 3n$ (c) $n^2 + 4n$ (d) $n^2 - 4n - 5$ (e) $n^2 + 2n - 8$

28. A certain investment has a fixed rate of interest and is compounded daily. If the value of the investment doubles in 10 years, the total number of years for the original investment to triple in value is best approximated by which of the following (log base may be any positive number)?
(a) $\frac{3\log 10}{\log 2}$ (b) $\frac{10\log 4}{\log 3}$ (c) $\frac{10\log 3}{\log 2}$ (d) $\frac{\log 30}{\log 3}$ (e) $\frac{15\log 10}{2\log 3}$

29.. What is the remainder of $S/5$ if $S = 1! + 2! + 3! + \ldots + 99!$
(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

30. In the figure, *ABCD* is a rectangle and *AE* is perpendicular to *BC*. The length of *BE* is

(a) $8/\sqrt{13}$ (b) 3/2 (c) $\sqrt{6}$ (d) $\sqrt{17/2}$ (e) $\sqrt{19/3}$