David Essner Exam 9 1989-1990

1. Given a set of three numbers, if the average of all three is 80, the averge of the first two is 76, and the first is 10 more than the second, then the smallest number of the three is

(a) 71 (b) 73 (c) 75 (d) 77 (e) 79

2. If $x = 1 - i$, where $i^2 = -1$, is a root of a polynomial $P(x) = 0$ with real number coefficients then $P(x)$ is divisible by

(a) $x^2 - 2x + 2$ (b) $x^2 + x + 1$ (c) $x^2 - 2x - 4$ (d) $x^2 - x + 1$ (e) $x^2 + 1$

3. If the integers m,n follow the pattern in the sequence

1,3,2,6,4,10,7,15,11,21,m,n then $m + n =$

(a) 36 (b) 38 (c) 40 (d) 42 (e) 44

4. Let k be a positive integer and S the sum of k successive positive integers. Then S is divisible by k

(a) always (b) never (c) if and only if k is even (d) if and only if k is odd

(e) none of (a)-(d)

5. Jack's current payroll deductions are 30% of his salary. If his current salary is increased by 10% and his current payroll deductions are increased by 15% then the percent increase in the net amount of his paycheck will be nearest what integer?

(a) 4 (b) 5 (c) 6 (d) 8 (e) 12

6. Eight baseball teams play in a double elimination tournament. (a team is eliminated when it has lost two games; all but one team is eliminated). The maximum number of games in the tournament is

(a) 15 (b) 18 (c) 21 (d) 24 (e) 26

7. If g is the greatest common divisor of 8547 and 4810 then the sum of the digits of g is

(a) 1 (b) 5 (c) 7 (d) 10 (e) 13

8. Given an isosceles triangle with area 1 and one angle 120°, the area of the triangle formed by joining the midpoints of the sides is

(a) 1/2 (b) 1/3 (c) 1/4 (d) $\sqrt{2}/4$ (e) $\sqrt{3}/4$

9. Bill and John made a series of bets, each bet $1 more than the previous bet. Bill won the first 30 bets, John won the last 20 bets and they broke even. The amount of the first bet was

(a) $0.40 (b) $2.25 (c) $9.50 (d) $15.75 (e) $35.50

10. Given (I) if $x > 3$ then $y < 7$ and (II) either $x < 2$ or $y > 9$ then

(a) $x < 2$ (b) $y \geq 7$ (c) $x \leq 3$ (d) $x > y$ (e) none of (a)-(d)

11. How many integers between 1 and 100 can be written as a product of two different prime numbers?

(a) 15 (b) 19 (c) 22 (d) 26 (e) 30

12. Given that the sum of the first and third term of a geometric series is 2, and the second term is 1 less than the first term, then the sum of all possible values for the first term is

(a) 0 (b) 2 (c) $\sqrt{5}$ (d) $2 + \sqrt{6}$ (e) $\sqrt{3} - 1$

13. If $a/b < c/d$ where a,b,c,d are positive numbers , then which number must be between a/b and c/d?

(a) $\frac{c/d - a/b}{2}$ (b) $\frac{a+c}{2(b+d)}$ (c) $\frac{a+d}{2(b+c)}$ (d) $\frac{a+3c}{b+3d}$ (e) $\frac{a+2d}{b+2c}$

14. If the three lines $ax + by = c$, $dx + ey = 0$, and $x + y = 1$ all have a common point of intersection then

(a) $ad + be = c$ (b) $ae + bd = c$ (c) $dc - db + ea - ec = 0$

(d) $da - dc + ec - eb = 0$ (e) $c(d + e) = a + b$

15. If $\cos(x - y) = \sqrt{6}/3$ and $\tan y = \sqrt{2}/2$, $0 < x,y < \pi/2$ then $\cos x =$

(a) 2/3 (b) $\sqrt{2}/5$ (c) $\sqrt{6}/8$ (d) $\sqrt{3}/2$ (e) 1/3

16. Three dice are rolled. the probability that exactly two different numbers occur is

(a) 1/3 (b) 5/12 (c) 5/9 (d) 11/16 (e) 71/216

17. John walked tree times from town A to town B. If his average speed for the first time was r mph (miles per hour), for the second time was 2r mph, and for the third time was 3r mph then his average speed in mph for the three trips was

(a) 2r (b) 4r/3 (c) 11r/6 (d) 18r/11 (e) 7r/3

18. Starting with a square of area 1, a circle is inscribed in the square and a square is then inscribed in the circle; a circle is then inscribed in the resulting square and a square in the circle and the process is repeated for a total of 5 pairs of inscribings. The area of the final square is

(a) $\sqrt{2}/48$ (b) $\pi/96$ (c) $\sqrt{924}$ (d) $\sqrt{5}/64$ (e) 1/32

19. If the integer w satisfies the equation $\log_2(\log_3(\log_2 w)) = 1$ then the sum of the digits of w is

(a) 4 (b) 6 (c) 8 (d) 10 (e) 12

20. The graph of the equation $y = 3x + 5$ is shifted 2 units to the right and 4 units up; it is then rotated 90° counterclockwise about the origin. The equation of the resulting graph is

(a) $x + 3y - 3 = 0$ (b) $2y = 5x + 9$ (c) $2x + 5y = 3$ (d) $4x + 9y = 7$
(e) $3x + 9y + 7 = 0$

21. An amount of money is invested at an annual rate of r compounded 4 times per year. At what annual rate compounded 2 times per year would the same investment produce the same interest?
(a) $2r$ (b) $r + r^2 - r^4$ (c) $r + r^2/8$ (d) $r + r^2 + r^3/4$ (e) $42r/41$

22. An urn contains 10 balls numbered 0,1,2,3,4,5,6,7,8,9. Three balls are drawn from the urn (no balls are returned to the urn). The probability the sum of the three numbers on the balls drawn is less than 9 is
(a) 1/7   (b) 2/15   (c) 193/720   (d) 11/60   (e) 29/120

23. The ratio $3^{100}/10^x$ is a number between 1 and 10 if $x =$
(a) 17   (b) 29   (c)47   (d)59   (e)78

24. Let $f(n)$ be a sequence of numbers defined by:
$f(1) = 1$ and $f(n) = f(n - 1) + 2$ if n is even and $f(n) = f(n - 1)/2$ if n is odd.
Then $4 - f(100) =$
(a) $(1/2)^{100}$ (b) 2^{98} (c) $(1/2)^{98}$ (d) $(1/2)^{49}$ (e) $(1/4)^{29}$

25. For what value of $a > 1$ do the two tangent lines from the point $(a,0)$ to the circle $x^2 + y^2 = 1$ meet at right angles?
(a) 2   (b) $\sqrt{2}$ (c) 3/2   (d) $\sqrt{3}$ (e) $\sqrt{3}/2$

26 Container A has two gallons of r% solution and container B has 2 gallons of s% solution. One gallon is taken from A and poured into B; one gallon is then taken from B and poured into A. The percent solution in A is then
(a) $(2r + s)/3$ (b) $(r + s)/2$ (c) $(3s + 2r)/5$ (d) $(3r + 2s)/5$ (e) $(5r + 6s)/11$

27. For $f(x)$ a real valued function define
$f^2(x) = f(f(x))$ and $f^{n+1}(x) = f(f^n(x))$ for $n = 2,3,\ldots$.
If $f(x) = x + 3$ then $f^{10}(x) =$
(a) $10x + 30$ (b) $x + 30$ (c) $10x + 3$ (d) $x^{10} + 3^{10}$ (e) $x^{10} + 30$

28. Given three real numbers x,y,z if $x + y - z$ is a negative number large in magnitude, $x - y - z$ is a very small positive number and $x + y + z = 1$ then
(a) $y > x > 0$ (b) $x > 0 > y$ (c) $z > y > x$ (d) $z > y > 0$ (e) $z > 0 > x$

29. The value $(10^{10} + 1)^{10} - 10^{100}$ is best approximated by 10^x where $x =$
(a) 91   (b) 101   (c) 1001   (d) 11   (e) 19

30. In the expansion of $(1 + a + a^2 + a^3 + a^4)^{10}$ the coefficient of a^3 is
(a) 48   (b) 64   (c) 81   (d) 127   (e) 220