David Essner Exam 10   1990-1991

1. Mary averaged 66% on three mathematics exams, 80 % on two English exams, and 91% on four history exams. Her average on all 9 exams is nearest the integer
        (a) 72   (b) 78   (c) 80   (d) 82   (e) 84

2. Two red dice and two green dice are rolled. The probability the sum on the red dice equals the sum on the green dice is
        (a) 73/648   (b) 1/6   (c) 7/36   (d) 55/216   (e) 1/2

3. Given the system of equations: $2x - y + z = 0$
                                     $x + 3y - z = 2$
                                     $x + 10y - 4z = 6$
if $x = a, y = b, z = c$ is a solution and $a + b = 1/7$ then $c =$
        (a) -5   (b) 3   (c) 3/7   (d) 55/216   (e) 23/7

4. The sum of the roots of the equation $x^5 - x^4 + 2x^3 - 2x^2 + x - 1 = 0$ is
        (a) 0   (b) 1   (c) 2   (d) 5   (e) 9

5. If $2^{4\log_2 x} = 3x^2$ then $x =$
            (a) $4\sqrt{3}$ (b) 3   (c) $\sqrt{3}$ (d) $\sqrt{42}$ (e) $\sqrt{6}/3$

6. How many integers between 1 and 1000 have the sum of the digits equal to 5?
        (a) 48   (b) 39   (c) 33   (d) 25   (e) 21

7. If p and q are logical statements then which of the following is always t rue?
        (a) if (p or q) then (if p then q)   (b) if (p and not q) then q (c) if p then (p and q)
        (d) if (q or p) then q (e) if (if p then q) then (p or q)

8. For positive integers by 'a mod $b \equiv c$' is meant the division of a by b gives a remainder c. Given x mod $7 \equiv 2$, y mod $7 \equiv 5$, and z mod $7 \equiv 3$ then xyz mod $7 \equiv$
        (a) 0   (b) 1   (c) 2   (d) 4   (e) 5

9. If a is a very small number then $4\sqrt{4+a}$ - $(8 + a)$ is best approximated by
        (a) $-a^3/16$ (b) $a^2/64$ (c) $a^3/27$ (d) $-a^2/8$ (e) $3a/329$

10. If an investment is compounded annually for 10 years at a rate r, then the value of the investment will triple if $r + 1 =$
        (a) $\log_{10}3$ (b) $10^{1/3}$ (c) $3^{1/10}$ (d) $\log_3 10$ (e) $1/10^{1/3}$

11. A department store purchases an item for $\$x$. There was markup of 50% to determine the original selling price, and then a 10% reduction of this original selling price to give the sales price of the item. The sales price of the item was then (in dollars)
        (a) $1.4x$ (b) $1.6x$ (c) $1.3x$ (d) $1.35x$ (e) $1.45x$

12. Tom and Bill each walked 5 miles. Tom walked 4/5 as fast as Bill, and the sum of the walking times of Tom and Bill was 2 hours. How fast, in miles per hour, did Bill walk?

     (a) 45/8 (b) 53/10 (c) 28/5 (d) 6 (e) 101/16

13. If $f(0) = 0$, $f(1) = 1$, and $f(n) = (f(n - 1) + \dfrac{f(n-2)}{2}$ for n > 1 then f(101) - 2/3 =

     (a) 5/202 (b) 3/101 (c) $1/3^{200}$ (d) $1/2^{101}$ (e) $-1/100^2$

14. Given a sequence of ten numbers, if the first number is 2 and each other number is the square of the preceding number, then the tenth number is

     (a) between 10 and 10^5 (b) between 10^5 and 10^{10} (c) between 10^{10} and 10^{50}
     (d) between 10^{50} and 10^{100} (e) more than 10^{100}

15. Jar *I* has 10 pounds of 20% solution and Jar *II* has 10 pounds of 50% solution. If *x* pounds is poured into Jar *I* from Jar *II* and then *x* pounds from Jar *II* into Jar *I,* the resulting mixture in Jar *I* is a 30% solution. Then *x* =

     (a) 16/3 (b) 21/4 (c) 9/2 (d) 5 (e) 7

16. Given triangle *ABC*, if $\angle A = 30°$, $\angle B = 105°$ and the altitude from *B* to side *AC* has length 2, then the area of the triangle is approximately

     (a) 4.18 (b) 5.46 (c) 4.74 (d) 6.12
     (e) cannot determine from the given information

17. Given an arithmetic progression, if the sum of the first ten terms is 205 and the fourth term is 16 then the first term is

     (a) 4 (b) 5 (c) 6 (d) 7 (e) 8

18. Bill and Tom make a series of bets. The first bet is $1 and thereafter each time Tom wins the bet, it is doubled, and each time Bill wins the bet it is reduced by $1. After 3 bets the maximum possible value of Bill's net winnings is

     (a) $(-1) (b) $2 (c) $1 (d) $0 (e) $5

19. If *n* is a very large number then the number $\dfrac{8^n + 7^n + 6^n}{9^n - 8^n - 7^n}$ is

     (a) near 0 (b) near 1 (c) between 2 and 10 (d) a very large number
     (e) none of (a)-(d)

20. For $a > 0$ the slope of the line through the point (0,-a) which intersects the parabola $y = x^2$ in exactly one point is

     (a) a^2 (b) $2a$ (c) $a^3/3$ (d) $a/2$ (e) $2\sqrt{a}$

21. There are how many ordered pairs of different integers betwen 1 and 10 inclusive such that 3 divides the least common multiple of the pair?

     (a) 16 (b) 34 (c) 40 (d) 48 (e) 72

22. If $0 < x < y^2 < z^3 < 1$ then which of the following must be true?
(I) $x^2 < y^3$ (II) $xz < y^3$ (c) $x + y < 3z$ (d) (IV) $xyz < z - x$
(a) III and IV   (b) I, II, and IV   (c) II and III   (d) I and II   (e) I and III

23. If $x^2 + y^2 = z^2$ where x, y, z are positive integers and in arithmetic progression then
(a) $z = 8x/3$ (b) $x + z$ is even   (c) $y - x$ is odd   (d) $x + y + z = 12$
(e) x is a prime number

24. The equations $x^2 + y^2 - 4y = 0$
$x^2 + y^2 - 8x - 10y = k$
have exactly one simultaneous solution for x and y if $k =$
(a) -32   (b) -16   (c) $\sqrt{13}$ (d) $3 + \sqrt{2}$ (e) 15

25. Given a triangle with vertices A, B, C, if $\angle C = 30°$, side BC has length 10 and side AB has length 4 then side AC has length
(a) $2\sqrt{21}$ (b) $\sqrt{58}$ (c) $\sqrt{29/3}$ (d) there are two possible values
(e) there are no possible values

26. Let $ABCD$ be a quadrilateral and let E be a point on side BC and F a point on side AD so that AB, CD and EF are parallel. If $BC = 12$, $AF = 1$ and $BE = FD = x$ then $x =$
(a) $\sqrt{10}$ (b) $2 + \sqrt{2}$ (c) $1 + \sqrt{3}$ (d) 7/2   (e) 3

27. If the hyperbola $xy = 1$ is shifted 3 units to the right and then rotated 90° clockwise about the origin then the equation of the resulting hyperbola is
(a) $xy - 3x = 0$ (b) $xy + 3y = 1$ (c) $xy + 3x = -1$ (d) $3xy + x = 1$
(e) $xy + y = -3$

28. In the major league world series two teams play until one teams wins 4 games; that team is then the winner. If both teams are equally likely to win each game, what is the probability the series will end in exactly 5 games?
(a) 3/16   (b) 1/4   (c) 5/32   (d) 1/8   (e) 1/5

29. In the Florida state lottery six numbers are selected from the integers 1 through 49 to give the winning combination. Contestants select six numbers and if they pick the winning combination then they are a winner. Cash awards are also given when the selection includes exactly five of the winning combination. If w is the probability of selecting all six winning numbers and s is the probability of selecting exactly five winning numbers then $s/w =$
(a) 258   (b) 986   (c) 664   (d) 1468   (e) 64

30. If $y = x - \sqrt{x^2 - 1}$, $x \geq 1$ then
(a) the smallest value of y is negative   (b) the largest value of y is greater than 1

(c) *y* does not have a largest value   (d) *y* does not have a smallest value
(e) none of (a)-(d)