David Essner Exam 11   1991-1992

1. A student scored 90% on a 200 point final exam, averaged 70% on four 100 point tests, and averaged 80% on numerous 50 point quizzes. If the combined average was 78%, how many quizzes were there?

(a) 7   (b) 8   (c) 9   (d) 10   (e) 11

2. The ratio of boys to girls at a certain school is 5 to 3. If the number of boys is increased by 20% and the number of girls increased by 30% then the resulting ratio of boys to girls is

(a) 20 to 13   (b) 18 to 11   (c) 16 to 9   (d) 24 to 17
(e) dependent on the original number of boys and girls.

3. A die is rolled four times. What is the probability that no number other than a 2 or 5 comes up?

(a) 5/216   (b) 7/36   (c) 1/81   (d) 17/1296   (e) 3/64

4. The negation of "no number in the interval $[a,b]$ is greater than both y and z" is equivalent to

(a) $y > b$ and $z > b$ (b) either $y > b$ or $z < a$ (c) either $y < a$ or $z < a$
(d) $b > y$ and $b > z$ (e) $y < a$ and $z < a$

5. The system of equations:

$$
\begin{aligned}
w + x &= a \\
x + y &= b \\
y + z &= c \\
w + z &= d
\end{aligned}
$$

has infinitely many solutions for w,x,y,z if

(a) $a + c = b + d$ (b) $c = b + d$ (c) $d = a - c$ (d) $a = b + c + d$
(e) $a - c = b - d$

6. Tom and Bill ran a foot race over the same distance. Tom ran the first half of the race at a rate r and the second half at $3/4\ r$. Bill ran the first half at a rate s and the second half at $5/4\ s$. If they finished in a tie then r/s =

(a) 1   (b) 3/5   (c) 15/16   (d) 35/27   (e) 16/15.

7. Tom has \$3 and Bill has \$2. They make a series of bets beginning with \$1, the bet doubling each time until one of them loses and has no more money or not enough to pay off the bet. If they have equal probability of winning each bet, the probability that Bill loses all his money is

(a) 3/5   (b) 6/16   (c) 1/2   (d) 5/8   (e) 4/7

8. A bottle contains 10 ounces of a 10% solution of alcohol in water. One ounce of alcohol is added to the solution and then one ounce of solution is removed. This is repeated one time. The result is an x% solution of alcohol in water where x is nearest the integer

(a) 20   (b) 22   (c) 24   (d) 26   (e) 28

9. If x,y,z are (positive) prime numbers then $x + y + z$

(a) must be even (b) must be odd (c) must be a prime number
(d) cannot be a prime number (e) none of (a)-(d)

10. John writes an integer between 1 and 100 on a piece of paper and asks Mary to guess the number. John replies to each guess that it is too high, too low, or correct and Mary then makes another guess unless she guessed correctly. If Mary chooses a good strategy then the least number of guesses and replies after which she will be sure to know John's number is

(a) 4 (b) 6 (c) 9 (d) 12 (e) 15

11. How many integers between 1 and 500 are both a perfect square and the sum of three consecutive integers?

(a) none (b) 1 (c) 4 (d) 7 (e) 10

12. Given a triangle with vertices A,B,C if $\cos A = \cos B = 3/5$ and the altitude to side c (opposite angle C) has length 2 then the area of the triangle is

(a) 3 (b) 5/2 (c) 8/3 (d) 4 (e) 10/3

13. Let $ABCD$ be a square inscribed in a circle of radius 1, and let E be the midpoint of side AB. The area of triangle CDE is

(a) 1 (b) 2/3 (c) $\sqrt{2}/2$ (d) $\sqrt{3}/2$ (e) $\pi/3$

14. The circle $(x - 1)^2 + y^2 = r^2$ lies inside the parabola $x = y^2$ if and only if r^2 is less than

(a) 4/9 (b) 9/25 (c) 2/3 (d) 1/2 (e) 3/4

15. If x,y,z are all positive numbers and $xy = xz = yz = 3$ then $\log_3 xyz =$

(a) 9 (b) 2/9 (c) 1/3 (d) 27 (e) 3/2

16. If $x = .0101$ and $y = -9{,}987$ then $\frac{3x+4y}{7x-2y}$ is

(a) near 0 (b) near -2 (c) near 3/7 (d) a very large positive number
(e) a negative number large in magnitude

17. The equation x + y + z = 10, where x,y,z are positive integers, has how many solutions for x,y,z?

(a) 16 (b) 28 (c) 36 (d) 48 (e) 64

18. Given a circle of radius 1, if x is the length of a chord subtended by an acute angle A then $x^2 =$

(a) sin 2A (b) $2(1 - \cos A)$ (c) $\cos(\pi/2 - A)$ (d) $\tan(A/2)$ (e) $2 \sin (A/2)$

19. The three digit number *ABC* to the number base 6 is necessarily divisible by 4 if
(a) *B*,*C* are both even (b) *A*,*C* are both even (c) *B* is odd and *C* is even
(d) *A* is odd and *B* is even (e) none of (a)-(d)

20. An investment, made at an annual rate of 8% compounded 4 times per year, would yield the same interest over a one year period if there was no compounding and the interest rate was nearest
(a) 8.04% (b) 8.08% (c) 8.16% (d) 8.2% (e) 8.24%

21. Suppose $f(x) = x^3 + Ax^2 + \text{B}x + C$ satisfies f(1) = 6 and has 2 as one root and 1 as the sum of the other two roots. Then $A + B + C =$
(a) -3 (b) 0 (c) 2 (d) 5 (e) 8

22. If $|x - y| \leq 2$ and $|x + y| \leq 2$ then the largest possible value of xy is
(a) 1/2 (b) $\sqrt{2}/2$ (c) 1 (d) $\sqrt{2}$ (e) 2

23. The number $|1.001^{1/3} - 3.001/300|$ is best approximated by
(a) 7/30,000 (b) 1/500,000 (c) 2/900,000 (d) 1/301,000 (e) 3/280,000

24. If $(M^2 - \text{N}^2) \equiv 1 \bmod 10$, where $M > N > 1$, then N cannot be
(a) 2 (b) 3 (c) 4 (d) 5 (e) 7

25. Let a_n be a sequence of numbers such that $a_0 = 1$, $a_n = a_{n-1} + 2$
if n is odd and $a_n = a_{n-1}/2$ if n is even. Then
$2^{50}\ (a_{101} - a_{99}) =$

(a) 1/2 (b) 1 (c) 3/2 (d) 5/2 (e) 4

26. The equations $y = |x - 2| + 3$ and $|y - 2| = x - 5$ have how many simultaneous solutions for x,y?
(a) none (b) 1 (c) 2 (d) 3 (e) 4

27. There are how many geometric sequences $a_1, a_2, a_3, \ldots$ such that each a_n is the square of an integer?
(a) none (b) 1 (c) 2 (d) a finite number more than 2 (e) infinitely many

28. If n is an integer, which number could divide both $3n + 17$ and $2n - 8$?
(a) 29 (b) 25 (c) 9 (d) 7 (e) 4

29. If c is a real number the sum of the roots of the equation $y^3 - (c^2 + 1)y + c = 0$ is

(a) 0 (b) $\frac{c-1}{4}$ (c) $\sqrt{c^2-1}$ (d) $\frac{c^3-1}{2}$ (e) none of (a)-(d)

30. Let S = $1/x - 1/x^2 + 1/x^3$. For which values of x, $x > 0$, is $S \geq 1/x^2$?

(a) $x \leq 1$ (b) $x \geq 1$ (c) $x = 1$ (d) all x (e) none of (a)-(d)

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