David Essner Exam 12 1992-1993

1. The sum of the last 5 digits of 25! (= 1x2x3x...x25) is

(a) 25 (b) 20 (c) 10 (d) 5 (e) 0

2. A green die and a red die are rolled. What is the probability the number on the green die is greater than the number on the red die?

(a) 1/2 (b) 1/3 (c) 17/36 (d) 4/9 (e) 5/12

3. Given three numbers, the average of the first two is 16, of the first and third is 12 and of the second and twice the third is 19. The average of the three numbers is

(a) 13 (b) 14 (c) 15 (d) 16 (e) 17

4. Gloria's Floral Shop now makes a profit of $10 on each dozen roses. If the price is reduced by 10% then they will make a profit of $8. What would be the price of the roses after the 10% reduction?

(a) $ 14 (b) $16 (c) $18 (d) $20 (e) $22

5. The statements $x \geq 3$ and (if $y \geq 7$ then $x < 3$) imply

(a) $y < 7$ and $x \geq 3$ (b) $x + y > 10$ (c) $y < 3$ or $x > 7$ (d) $y \leq 7$ and $x > 3$
(e) $y = 7$ or $x = 3$.

6. The system of equations

$$\begin{aligned} x - 2y + 3z &= a \\ 3x + y + 4z &= a \\ 5x - 3y + 10z &= b \end{aligned}$$

can be solved for x,y,z provided

(a) $a = 2b$ (b) $b = 3a$ (c) $b = 2a$ (d) $a = 3b$ (e) $a = b$.

7. Jar A has 10 ounces of a 20% solution and jar B has 30 ounces of a 10% solution. Five ounces from jar A and x ounces from jar B are poured into an empty jar to give a 12% solution. Then x =

(a) 12 (b) 15 (c) 35/2 (d) (e) 45/2

8. John and Bill run a 1,000 foot race. Bill runs at a constant speed. John runs at .8 the speed of Bill for the first 600 feet and x times the speed of Bill for the rest of the race. If they end in a tie then x =

(a) 4/3 (b) 5/3 (c) 5/4 (d) 8/7 (e) 8/5

9. Tom and Jack play a series of games. Initially Jack has $5 and Tom has $2. For each game if Jack loses he gives Tom $2 (or $1 if it is his last dollar); if Tom loses he gives Jack $1. If Tom gets Jack's last dollar(s) after n games, then n cannot be

(a) 12 (b) 13 (c) 14 (d) 15 (e) 16.

10. If x is a large positive number then $\dfrac{\log_{10} x}{x}$ is

(a) near 0 and positive (b) a large positive number
(c) a very large negative number (d) near 1 (e) near 0 and negative.

11. How many integers between 1 and 500 are both a perfect square and the sum of three consecutive even integers?

(a) none (b) 1 (c) 3 (d) 6 (e) 10

12. If a sum of money is invested at a rate r compounded 2 times per year then in order that the sum double in one year r must equal

(a) .75 (b) .5 (c) $\sqrt{2}/2$ (d) $2(\sqrt{2} - 1)$ (e) $\frac{\sqrt{2}+1}{4}$

13. Suppose the Yankees and Dodgers play 7 games, with one team winning 4 and the other 3; the team that wins 4 wins the last game. In how many ways can this happen?

(a) 40 (b) 56 (c) 70 (d) 84 (e) 124

14. If $\log_{10} b = c$ and $\log_b c = d$ then $d =$

(a) 10^c (b) $c \log_{10} c$ (c) $\log_{10}(1/c)$ (d) $\frac{\log_{10} c}{c}$ (e) $10^{1/c}$

15. Given that $4x^3 + 8x^2 - 11x - 15$ has $x = 3/2$ for one root, then the sum of all its roots is

(a) -7/2 (b) -2 (c) 0 (d) 9/2 (e) 6

16. The value $(1.01)^{10}$ is best approximated by

(a) 1.10462 (b) 1.10512 (c) 1.10396 (d) 1.10628 (e) 1.10814

17. If $a_0 = 1$, $a_n = \frac{a_{n-1}}{3}$ for n odd and $a_n = a_{n-1} + 1$ for n even then if n is a large number the product of a_n and a_{n+1} is near

(a) 1 (b) 2/3 (c) 3/4 (d) 5/3 (e) 5/4.

18. Given triangle ABC has area 1, $\angle A = 45°$, and $\angle B = 30°$, if x is the length of the altitude to side AB then $x^2 =$

(a) $\sqrt{6}/4$ (b) $\frac{\sqrt{3}+\sqrt{2}}{4}$ (c) $1/\sqrt{3} + 1/\sqrt{2}$ (d) $2/\sqrt{6}$ (e) $\frac{2}{\sqrt{3}+1}$

19. Given triangle ABC with $\angle C = 90°$, let D be a point on AB and E a point on AC so that DE is parallel to BC. If $AD = EC = 2$ and $AE = 1$ then the area of the triangle is

(a) $9\sqrt{3}/2$ (b) $5\sqrt{2}$ (c) 6 (d) $3 + \sqrt{2}$ (e) $2 + \sqrt{3}$

20. The product of the x coordinates of all points which lie on both of the graphs of $|x - y| = 3$ and $x^2 + y^2 = 29$ is

(a) 0 (b) 87 (c) 100 (d) 7569 (e) -87

21. If $(M + 2N) \equiv 4$ Mod 6 and $(2M + N) \equiv x$ Mod 3, then x could be which of the following numbers?

(a) 30 (b) 32 (c) 34 (d) 36 (e) 40

22. The population of a large country has a continuous growth rate so as to double every 12 years. If x is the population in 1992 then the population in 1996 will be

(a) $4x/3$ (b) $\dfrac{4x}{\log_2 12}$ (c) $\log_{12}(4x)$ (d) $(2)^{1/3}x$ (e) $(4)^{1/12}x$

23. Given a circle of radius 8, what is the largest odd integer which is less than the largest odd integer which is less than the ratio of the circumference to the diameter of the circle?

(a) 1 (b) 3 (c) 5 (d) 7 (e) none of (a),(b),(c),(d).

24. The parabola $y = x^2$ is tangent to the circle $x^2 + (y - 1)^2 = C$ if $C =$

(a) 1 (b) 4/5 (c) 3/4 (d) 2/3 (e) 1/2.

25. Which number is the largest?

(a) 30^{20} (b) 4^{100} (c) $\log_2 1{,}000{,}000{,}000{,}000$ (d) $999{,}999^3$ (e) 30!

26. If $S_n = 1 + 2 + 2^2 + 2^3 + ... + 2^n$ then the smallest integer n for which $S_n > 50{,}000$ is

(a) 9 (b) 15 (c) 22 (d) 42 (e) 56

27. If x,y,z are positive numbers such that $3 < xy < 4$, $4 < xz < 5$, $5 < yz < 6$ then

(a) $x > \sqrt{2}$ (b) $y > 5/2$ (c) $z < \sqrt{3}$ (d) $y < \sqrt{2}$ (e) $x < 3/2$.

28. The sum of the digits of the largest prime number which divides the integer 14,280 is

(a) 6 (b) 8 (c) 11 (d) 13 (e) 15

29. The units (last) digit of 7^{50} is

(a) 1 (b) 3 (c) 5 (d) 7 (e) 9.

30. Let $ABCD$ be a parallelogram with area 1 and let E be a point on side CD. If triangle BCE has area 1/6, side AD has length 2, and segment ED has length 1 then the length of CD is

(a) 3 (b) 4/3 (c) 2 (d) 6/5 (e) 3/2

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