**David Essner Exam 13    1993-1994**

1. A student took four exams, each time doubling the score of the previous exam. If the average of the four exams was $x$ then the score on the first exam was $cx$ where $c$ =

(a) 1/4   (b) 2/9   (c) 4/15   (d) 3/16   (e) 7/24

2. If $x = 6 \sin \pi(3t + 4)$ is the $x$ coordinate of a point at time $t$ minutes, then $x$ has value 0 every $s$ seconds where $s$ =

(a) 10   (b) 15   (c) 20   (d) 30   (e) $\pi/6$.

3. Let $a,b,c$ be sides of a triangle with respective opposite angles $A,B,C$. If $a = 10$, $b = 6$ then there is exactly one triangle with

(a) $c = 3$   (b) $A = \pi/6$   (c) $B = \pi/3$   (d) $C = \pi/4$   (e) none of (a)-(d)

4. For real numbers $a,b$ let $A$ be the statement $ab \geq 0$ and $B$ the statement $|a + b| = |a| + |b|$. Let $C$ be the statement '$A$ implies $B$' and $D$ the statement '$B$ implies $A$'. Then which of the following is true?

(a) both $C$ and $D$   (b) neither $C$ nor $D$   (c) $C$ but not $D$

(d) $D$ but not $C$   (e) none of (a)-(d).

5. Given the parabola $y = x^2 + ax - a^2$ the $y$ coordinate of the vertex is

(a) $3a/2$   (b) $-2a/3$   (c) $a + 1$   (d) $-5a/4$   (e) $2a/3$

6. Given a triangle with length of sides 3,4,5 the radius of the circumscribed circle is

(a) $\sqrt{6}$   (b) 2   (c) 9/4   (d) 7/3   (e) 5/2.

7. Teams $A$ and $B$ play a sequence of games; each game the probability that $A$ wins is 2/3. What is the probability that $A$ will win 2 games before $B$ wins 2 games?

(a) 2/3   (b) 8/9   (c) 15/16   (d) 20/27   (e) 74/81

8. The difference of 100 and $\sqrt{100^2 - 1}$ is near $1/k$ where k =

(a) 10   (b) 200   (c) 500   (d) 4000   (e) 5500

9. On January 1 of two consecutive years a man invests \$100 at an annual rate of $r$ compounded 2 times per year. At the end of two years if $V$ is the total value of the investment then $V/100$ is approximately (assume $r$ is a small number):

(a) $3r + 2$   (b) $2r + 3$   (c) $2(r + 1)$   (d) $\frac{3r+1}{2}$   (e) $\frac{4r+1}{2}$

10. If w,x,y,z are positive integers which satisfy the equations

$$w = 3x + 2$$
$$x = 7y + 5$$
$$y = 8z + 3$$

then the sum of the digits of the smallest possible value of $w$ is

(a) 5   (b) 8   (c) 10   (d) 14   (e) 17

11. If $\log_a 16 = 3$ and $\log_b 4 = 7$ then $\log_a b =$

(a) 3/7 (b) 64/21 (c) 21/64 (d) 21 (e) 3/14

12. Jars A,B,C each have 100 grams of 20% solution and jar *D* has 40 grams of 5% solution. If *x* grams from A are put in *B*, then *x* grams from *B* are put in C and then x grams from *C* are put in *D*, the resulting solution in *D* is 10%. Then $x =$

(a) 20 (b) 24 (c) 82/3 (d) 65/4 (e) $\sqrt{426}$

13. Bill runs 100 yards at an average rate of *r* and another 100 yards at an average rate of *s*. His average rate for the 200 yards is

(a) $\frac{r+s}{2}$ (b) $\frac{rs}{25}$ (c) $\frac{200}{r+s}$ (d) $\frac{50(r+s)}{rs}$ (e) $\frac{2rs}{r+s}$

14. A store purchases 100 coats for \$5000. Suppose they sell *x* coats at \$80 each, then *y* coats reduced to \$40 each, and then *z* coats at a clearance price of \$20 each (all of the coats now being sold). If the sales of the coats exceeds the cost then

(a) $x + y > 120$ (b) $4x + y > 240$ (c) $x + 5y > 200$ (d) $3x + y > 150$
(e) $2x + 3y > 190$

15. The sum of the real roots of $x^3 + (\pi - 1)x^2 - \pi = 0$ is

(a) 0 (b) 1 (c) $\pi/2$ (d) $\pi - 1$ (e) $\pi + 1$

16. In the plane the three lines $y = 2x + 1$, $y = x + a$, $y = 3x + b$ have a single point of intersection provided

(a) $a + 3b = 2$ (b) $a + b = 2$ (c) $a + 2b = 3$ (c) $a - 3b = 2$ (e) $2a - b = 1$

17. The point on the line $y = 2x - 3$ nearest the point (2,5) is

(a) (3,3) (b) (3.2,3.4) (c) (3.6,4.2) (d) (3.8, 4.6) (e) (4,5)

18. John has \$2 and Bill has \$1. They make a series of bets, each bet for \$1 and each equally likely to win, until one of them has no money. The probability that John wins all the money is

(a) 1/2 (b) 2/3 (c) 3/5 (d) 11/16 (e) 5/6.

19. If $y = |3x - 1| - |2x - 5|$ then the (minimum) least value of *y* is

(a) -13/3 (b) -2 (c) -1 (d) 1/3 (e) 5/2

20. Given geometric sequences $a_1, a_2, \ldots$ and $b_1, b_2, \ldots$ if
$a_1 + b_1 = 3$, $a_1 b_2 = 1$, $a_2 b_1 = 4/3$, $a_1 b_3 = 1/2$ and $b_1 > a_1$ then $a_3 =$

(a) 3/4 (b) 1/8 (c) 3/8 (d) 9/2 (e) 4/9.

21. In how many ways can one select three integers from the set {0,1,2,...,9}? Repetitions are permitted and ordering is not relevant (e.g. 1,5,5 and 5,1,5 are the same selection).

(a) 64 (b) 160 (c) 220 (d) 360 (e) 520

22. Among the following integers which is the least value of $n$ such that $3^n > 2^{n+10}$ ?

(a) 8 (b) 12 (c) 15 (d) 18 (e) 24

23. Let $P(x) = (1 + 2x)^5$ and $Q(x) = 7x - 6x^3 + x^5$ . If $x$ is a large positive number then $\frac{P(x)}{Q(x)}$ is

(a) near 0 (b) near 32 (c) near 7 (d) near -42
(e) a large positive number

24. If an integer is initially assigned the value 1 and then n times is replaced by 1 more than twice its value, the resulting number is

(a) $2^{n+1} - 1$ (b) $2^n + n$ (c) $2^{n-1} + 3n$ (d) $2^n + 2n$ (e) $2^{2n+1}$

25. Suppose $P(x)$ is a polynomial with integer coefficients which when divided by $x^2 - x - 2$ gives a remainder of $3x - 11$ and when divided by $x^2 - 2x - 3$ gives a remainder of $(2x + t)$. Then $t$ =

(a) -2 (b) 0 (c) 3 (d) 7 (e) -12.

26. Circle $A$ has four times the area of circle $B$. If an arc of $\pi/6$ on circle $A$ has length $x$ then an arc of $\pi/4$ on circle $B$ has length $cx$ where c =

(a) 2/3 (b) 3/8 (c) 3/4 (d) 3/2 (e) 4/3.

27. If $d$ is the largest integer which leaves the same remainder when dividing into 983, 183, and 1333 then the sum of the digits of $d$ is

(a) 3 (b) 5 (c) 8 (d) 13 (e) 14.

28. What positive number is such that the reciprocal equals one less than the number?

(a) $(1 + \sqrt{2})/2$ (b) $(\sqrt{2} - 1)/4$ (c) $(1 + \sqrt{5})/2$ (d) $(\sqrt{3}+\sqrt{2})/4$
(e) $(\sqrt{2}+\sqrt{5})/2$

29. If $a = 0.999$ and $b = 1.001$ then which of the following numbers is the largest?

(a) $a^b$ (b) $b^a$ (c) $ab^2$ (d) $b/a$ (e) $\frac{a+b}{2}$

30. The least common multiple of the set of integers {1,2,3, 4,5,6,7,8,9,10} is

(a) 2520 (b) 1260 (c) 840 (d) 420 (e) 10!

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