David Essner Exam 15 1995-1996

1. On a class test consisting of 3 questions, 20% got 3 right, 40% got 2 right, and 30% got 1 right. The average number of right answers was

(a) 1.5 (b) 2.0 (c) 1.8 (d) 1.7 (e) depends on the class size

2. Of 20 students 14 take math, 8 do not take English, and 4 take English but do not take math. How many are there who take math but do not take English?

(a) none (b) 2 (c) 3 (d) 4 (e) 6

3. The lengths of each side of a cube are increased by 10%. By what percent is the volume then increased?

(a) 30 (b) 33.1 (c) 10 (d) 3 1/3 (e) 130

4. There are how many positive integers x such that $\frac{8x}{8+x}$ is an integer?

(a) none (b) 1 (c) 2 (d) 3 (e) more than 3

5. The volume of a sphere in cubic feet is the same as the surface area of the sphere in square feet. The radius of the sphere in feet is

(a) $2\pi/3$ (b) $\pi/2$ (c) 9 (d) 3π (e) 3

6. If the four equations:
$$x + y + z = a$$
$$x - y + z = b$$
$$x - y - z = c$$
$$3x - y + z = d$$
have a solution for x,y,z then

(a) $a + b - c - d = 0$ (b) $a + b + c - d = 0$ (c) $a - b + c - d = 0$
(d) $a - b - c + d = 0$ (e) $a + b - c + d = 0$

7. If an amount of money is invested at a rate of $100r$ percent compounded annually, in how many years will the investment double? (all logs to the base 10).

(a) $\frac{\log 2}{\log(1+r)}$ (b) $\frac{1}{2^r - 1}$ (c) $-\log(2^r - 1)$ (d) $\log\frac{r}{2r+1}$ (e) 2^{1-r}

8. John and Bill run a 1 mile race. Bill runs at a constant speed. If John runs 9/10 as fast as Bill for a distance D and 11/10 as fast as Bill for the remainder of the race then they will end in a tie if D is what part of a mile?

(a) 1/2 (b) 9/11 (c) 11/20 (d) 9/20 (e) 19/40

9. Three different numbers are selected at random from the set of integers 1 through 10. The probability that at least one of the integers is even is

(a) 2/3 (b) 5/6 (c) 7/8 (d) 11/12 (e) 15/16

10. The first term of an arithmetic progression is 3 and the sum of the first ten terms of the progression is 120. The fourth term of the progression is then

(a) 9 (b) 10 (c) 8 (d) 11 (e) 15/2

11. Given a triangle with vertices A,B,C if $\tan A = 2/3$, $\tan B = 3/4$ and the length of the altitude from C to side AB is 6 then the area of the triangle is

(a) 24 (b) 30 (c) 36 (d) 42 (e) 51

12. Ten pounds of a 10% solution is mixed with 20 pounds of a 20% solution, and the resulting mixture is added to x pounds of a 30% solution. If the final mixture is a 25% solution then $x =$

(a) 46 (b) 48 (c) 50 (d) 54 (e) 60

13.In the Cartesian plane the line through the point (1,2) and tangent to the circle $x^2 + y^2 = 1$ intersects the circle in the point

(a) (-3/5,4/5) (b) $(-1/\sqrt{2}\ ,1/\sqrt{2}\ )$ (c) $(-1/2,\ \sqrt{3}/2)$ (d) $(-1/3,\ 2\sqrt{2}/3)$
(e) (-4/5,3/5)

14. In the Cartesian plane if (a,b) is the point on the line $y = 5x$ which is nearest the point (2,3) then $a =$

(a) 2/3 (b) 17/26 (c) 3/4 (d) 11/19 (e) 1

15. If the equation $x^3 + ax^2 + bx + c = 0$ has a root $x = 1$, which expression must equal 0?

(a) $2a - b + c - 1$ (b) $a + b - 2c + 1$ (c) $a + b + c + 1$ (d) $a - b + c - 1$
(e) $a + 2b + c - 1$

16. If x is near 0 then $\dfrac{\sqrt{1+x}-1}{x}$ is near what number?

(a) 0 (b) 2/3 (c) 1 (d) 1/2 (e) /2

17. If x is a large number then $\sqrt{x^2+1}$ - x is best approximated by

(a) $\dfrac{2x-1}{4x^2}$ (b) $\dfrac{3x-2}{6x^2}$ (c) $\dfrac{4x^2-1}{8x^3}$ (d) $\dfrac{6x^2-2}{9x^3}$ (e) $\dfrac{8x^3-50}{12x^4}$

18. If 3 persons are selected at random, what is the probability they were born on different weekdays? (i.e. no two were born on the same weekday).

(a) 3/7 (b) 6/7 (c) 18/49 (d) 30/49 (e) 2/3

19. If $\frac{x^2+9x+7}{x^2(x+1)} = \frac{Ax+B}{x^2} + \frac{C}{x+1}$ for all numbers x other than 0 and -1 then $A + B + C =$

(a) 2 (b) 4 (c) 8 (d) 10 (e) there are no solution values for A,B,C.

20. Given two concentric circles of radius 1 and 2, if a tangent to the smaller circle intersects the larger circle in points A,B then the length of the segment AB is

(a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) $\sqrt{2}\,\pi$ (d) $3\pi/2$ (e) $\frac{1+\sqrt{3}}{2}$

21. The sum 7/360 + 5/756 equals

(a) 97/3600 (b) 197/7560 (c) 77/2520 (d) 147/5640 (e) 37/840

22. If $a > 1$ the sum of all solutions of $x^{\log_a x} = a^2 x$ is

(a) $1 + 1/a^2$ (b) $3a^2 - 1$ (c) $2a/3$ (d) $2 + 2^a$ (e) $\frac{a^3+1}{a}$

23. If $x(0) = 1$ and $x(n + 1) = \frac{x(n)}{10} + 1$ for $n = 0,1,2,3,...$ then for n a very large integer $x(n)$ is near

(a) 2 (b) 3/2 (c) 10/9 (d) 10 (e) infinity

24. If 3^{20} is divided by 11 the remainder is

(a) 1 (b) 3 (c) 5 (d) 7 (e) 9

25. Let x,y,z be positive integers such that x is odd but not both y and z are odd. Of the three statements: (A) xy is even (B) $x + z$ is odd (C) $x + yz$ is odd exactly which ones must be true?

(a) A and B (b) B and C (c) A and C (d) B only (e) C only

26. The equation $xy = 1000$ has how many solution pairs (x,y) where x and y are positive integers (Note: (1,1000) and (1000,1) are different solution pairs).

(a) 12 (b) 13 (c) 14 (d) 15 (e) 16

27. Initially Tom has \$1 and Bill has \$2. They wage a series of \$1 bets until one of them has all the money i.e. the \$3. Each bet the person with \$2 has probability 1/3 of winning. What is the probability Bill wins all the money?

(a) 1/3 (b) 2/3 (c) 1/2 (d) 3/5 (e) 2/5

28. Given $0 < x < 1$ let $y = x^x$ and $z = x^y$. Then

(a) $x < y < z$ (b) $x < z < y$ (c) $y < x < z$ (d) $z < x < y$ (e) $y < z < x$

29. Given the complex number $z = 1 + i$, where $i^2 = -1$, then z^{10} =

(a) $10 + 10i$ (b) $2^{10}(1 + 2i)$ (c) $32i$ (d) $10 - 5i$ (e) 16

30. Given the values for a triangle: $\angle A = 30°$, side $b = 10$ and side $a = x$, where side a is opposite $\angle$ A, then the values of x for which there are 2 possible triangles is described by

(a) $5 < x < 10$ (b) $x < 5/2$ (c) $5/3 < x < 10$ (d) $5 < x < 5/3$
(e) $5/2 < x < 10$

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