David Essner Exam 17 1997-1998

1. A student has 4 regular exam grades of 90,80,90,and 60 percent. What percent score, which is counted twice in averaging the grades, must the student earn on a final exam in order to obtain an overall average of 85 percent?

(a) 95 (b) 88 (c) 91 (d) 93 (e) 85

2. The initial price of a gidget is increased by 10%, and the resulting price is then decreased by 10%. The final price is then what percent of the initial price?

(a) 99 (b) 100 (c) 101 (d) 100.9 (e) 99.1

3. A die is rolled 3 times. What is the probability a different number occurs each time?

(a) 1/2 (b) 5/9 (c) 1/3 (d) 7/18 (e) 5/12

4. The sum of all solutions of $|2x + 5| = |3x - 2|$ is

(a) 7 (b) 11/3 (c) 15/7 (d) -1/3 (e) 32/5

5. There are how many positive integer divisors of 1296 (= 34x42)?

(a) 16 (b) 22 (c) 25 (d) 28 (e) 32

6. Which statement is equivalent to "If $x \neq 3$ then $y > 7$" ?

(a) If $x > 7$ then $y \neq 3$ (b) If $y > 7$ then $x \neq 3$ (c) If $x = 3$ then $y = 7$
(d) If $y \leq 7$ then $x = 3$ (e) If $y \neq 3$ then $x > 7$

7. Let A,B be sets and ' denote the complement of a set. If $A \cup B$ has 25 elements, $A' \cap B$ has 12 elements and $A \cap B'$ has 10 elements then $A \cap B$ has how many elements?

(a) 3 (b) 13 (c) 15 (d) 17 (e) 2

8. The volume of a cubical box equals the total surface area. The length of the edges of the box is then

(a) 2 (b) 4 (c) 6 (d) 8 (e) 12

9. If $x(n) = \dfrac{x(n-1)}{2}$ - 1 and $x(0) = a$ then $x(6) = 0$ if a =

(a) 12 (b) 37/2 (c) 46 (d) 84 (e) 126.

10.Triangle ABC has area 10. If C is a right angle, $\angle A = 30°$, and h is the length of the altitude to side AB then h^2 =

(a) $12\sqrt{2}$ (b) $6\sqrt{2}$ (c) $8\sqrt{2}$ (d) $5\sqrt{3}$ (e) $6\sqrt{3}$

11. A person invested \$1 in 1897; the interest rate was 10% compounded annually. The dollar value of the investment in 1997 is between

(a) 10 and 100 (b) 100 and 400 (c) 400 and 1,000 (d) 1,000 and 4,000
(e) 4,000 and 16,000

12. If (a,b) is the center of the circle passing through the points (0,3), (-1,0) and (2,1) then $a + b =$

(a) 3/4 (b) 3/2 (c) 7/4 (d) 2 (e) 9/4

13. John and Bill run a 1 mile race, each at a constant rate. If John ran 7 miles per hour faster he would have run twice as fast as Bill. When John finished the race, Bill had run 4/5 mile. How fast in miles per hour did John run?

(a) 14 (b) 12 (c) 49/5 (d) 35/3 (e) 43/4

14. If x gallons of a 3% solution are mixed with y gallons of a 6% solution , and the result is added to 10 gallons of a 5% solution then the final result is a 4% solution . Then

(a) $2y - x = 10$ (b) $x = 2y + 10$ (c) $y = 2x + 10$ (d) $2x - y = 10$
(e) $x + 2y = 10$

15. Given that the equation $x^3 - 7x + 6 = 0$ has roots $x = 1$ and $x = 2$ then the third root is

(a) -3 (b) -5 (c) -1 (d) 3 (e) 5

16. If x is a number near 2 then $\dfrac{1/x - 1/2}{2 - x}$ is

(a) a large positive number (b) near -1 (c) near 0
(d) near 1/4 (e) near 1/8

17. Given that $\{a_k\}$ is a sequence of numbers and for each n,

$$a_1 + a_2 + ... + a_n = \frac{n}{n+1} \quad \text{then } a_6 =$$

(a) 6/7 (b) 5/6 (c) 1/7 (d) 1/12 (e) 1/42

18. In a circle of radius 1 let an equilateral triangle ABC be formed with A the center of the circle and B,C points on the circle. The area of the ice cream cone shaped region ABC is then

(a) $\sqrt{3}/2$ (b) $\sqrt{2}/2$ (c) $\pi/4$ (d) $\pi/6$ (d) $\pi/3$

19. A man standing on a railway bridge .6 of the distance from one end hears a train coming at a speed of 50 mph. If he then runs at a speed of r mph towards either end of the bridge then he and the train will reach that end of the bridge at the same instant . Then $r =$

(a) 10 (b) 8 1/3 (c) 9 2/3 (d) 8 (e) 11 1/4

20. If $y \leq 2 + x$, $3x + y \leq 4$ and $2y \geq x + 2$ then the smallest possible value of y is

(a) -3/2 (b) -1/2 (c) 0 (d) 4/3 (e) 1

21. If $y_0 = x$ and $y_{n+1} = y_n^2$ for $n = 1,2,...$ then $\log_{10} xy =$

(a) 100 (b) 1024 (c) 20 (d) 14,400 (e) 20,000

22. The coefficient of x^3 in the expansion of $(1 + 2x)^{12}$ is

(a) 640 (b) 1820 (c) 1440 (d) 1520 (e) 1760

23. For integers a,b,c let '$a \equiv b \bmod c$' mean that $\frac{a-b}{c}$ is an integer. If $x \equiv 5 \bmod 6$ and $y \equiv 2 \bmod 3$ then of the three statements

A: $xy \equiv 10 \bmod 3$; B: $xy \equiv 10 \bmod 6$; C: $xy \equiv 10 \bmod 2$

which one(s) must be true?

(a) A only (b) A and B only (c) A and C only (d) all three (e) none

24. How many combinations of 3 different positive integers (such as 3,4,13) are there which add to 20?

(a) 18 (b) 24 (c) 28 (d) 32 (e) 36

25. A geometric progression of positive numbers has 1 for the first term and 10/9 for the sum of the second and third terms. Then the difference of the second and third terms is

(a) 1/9 (b) 5/18 (c) 2/9 (d) 4/9 (e) 7/18

26. In a triangle with angles A,B,C and respective opposite sides a,b,c if $A = 120°$, $a = 10$ and $\cos B = 4/5$ then $b =$

(a) $5\sqrt{2}$ (b) 6 (c) $4\sqrt{3}$ (d) $3\sqrt{5}$ (e) $2\sqrt{10}$

27. If $w = w_0$, $x = x_0$, $y = y_0$, and $z = z_0$ satisfy the equations

$$\begin{aligned} w + x + y \quad &= 6 \\ y + z &= -1 \\ x + y \quad &= 1 \\ x + \quad z &= 4 \end{aligned}$$

then $w_0 + x_0 + y_0 + z_0 =$

(a) -2 (b) 2 (c) 3 (d) 5 (e) 7

28. If m and n are positive integers, $n \neq 1$, such that $1997 = 19m + 97n$ then $m + n =$

(a) 20 (b) 23 (c) 26 (d) 29 (e) 31

29. For which values of x is the expression $\frac{\sqrt{1-x}}{1-\sqrt{x}}$ a real number?

(a) $|x| < 1$ (b) $0 < x < 1$ (c) $0 < x \leq 1$ (d) $0 \leq x < 1$
(e) $0 \leq x \leq 1$

30. A box contains 8 red balls and 2 black balls. If three balls are simultaneously drawn from the box, what is the probability both black balls were drawn?

(a) 1/10 (b) 8/125 (c) 1/15 (d) 2/25 (e) 1/16