David Essner Exam 18 1998-1999

1. Let $x(n)$ be a sequence of numbers such that $x(1) = 2$, $x(2) = 3$ and $x(n+2) = x(n+1) + x(n)$ for $n \geq 1$. The smallest value of n such that $x(n) > 100$ is

(a) 6 (b) 8 (c) 10 (d) 12 (e) 14

2. Given the right triangle with vertices (0,0), (20,0), and (20,40) in the Cartesian plane, there are how many points (x,y) inside the triangle such that x and y are both integers? (Do not count points such as (0,0) and (1,2) on the boundary)

(a) 149 (b) 275 (c) 348 (d) 361 (e) 452

3. If a is a rational and b an irrational number then *ab*

(a) must be rational (b) must be irrational
(c) could be either irrational or rational
(d) could be neither rational nor irrational (e) none of (a)-(d).

4. If $x^2 + y = y^2 + x$ and $x \neq y$ then 1 =

(a) $x - y$ (b) $x + 2y$ (c) $2x + y$ (d) $y - x$ (e) $x + y$

5 A train goes from town *A* to town *B* in 9 hours. A second train goes from *B* to *A* in 6 hours. If the trains leave at the same time, go at a constant rate, and make no stops then how many hours after they leave will they meet?

(a) 2.8 (b) 3.2 (c) 3.6 (d) 3.9 (e) 4.2

6. If x,y,z are positive numbers and the ratio of z to x is the same as the ratio of x to y then if $x = 1$ when $z = 2$ it follows that if $z = 4$ then $x =$

(a) $\sqrt{2}$ (b) 8 (c) 1/2 (d) $2\sqrt{2}$ (e) 1/4

7. A student takes 10 exams and has an average of 76. If the two lowest scores of 46 and 52 are discarded then the average of the remaining 8 exams is nearest the integer

(a) 78 (b) 81 (c) 83 (d) 85 (e) 87

8. Bill runs 4/3 as fast as John. If John has a 5 minute head start how many minutes does it take for Bill to catch up with him:

(a) 6 2/3 (b) 12 (c) 15 (d) 20 (e) 24 1/3

9. What is the maximum value of $|x - 1| - |x^2 - 1|$ for all real number values of x?

(a) 1 (b) 2 (c) 3/2 (d) 5/2 (e) 3

10. The sum $\sum_{n=0}^{20}(n-7)$ =

(a) 57 (b) 60 (c) 63 (d) 66 (e) 69

11. The center of the circle that passes through the points (0,0), (0,2), (1,3) is

(a) (1,1) (b) (2,1) (c) (1,2) (d) (2,2) (e) (3,2)

12. For which polynomial is the sum of the roots (real or complex) equal to 4 and the product of the roots equal to 2?

(a) $x^3 - 4x^2 + 6x - 2$ (b) $x^3 + 4x^2 - 6x - 2$ (c) $x^3 + 6x^2 - 4x - 2$
(d) $x^3 - 6x^2 + 4x - 2$ (e) $x^3 - 2x^2 + 6x - 4$

13. A person invests an amount of money for one year at a simple annual rate of interest of 8% and the interest is taxed at a rate of 20% at the end of the year. If the interest were not taxed what simple annual rate of interest would have resulted in the same amount of money retained by the person?

(a) .4 (b) 2.5 (c) 4 (d) 5.6 (e) 6.4

14. If $\log_8 3 = p$ then $\log_3 2 =$

(a) $p/3$ (b) $3p/2$ (c) $3/p$ (d) $2p/3$ (e) $1/3p$

15. If $\log_3 7 = x$ and $\log_7 10 = y$ then $\log_3 10 =$

(a) $x + y$ (b) xy (c) $x - y$ (d) $7x/y$ (e) $7y/x$

16. If $f(x) = 3x + 1$ for $x < 0$ and $f(x) = 2x - 5$ for $x \geq 0$ then $f(f(2)) =$

(a) 7 (b) -1 (c) -2 (d) 22 (e) 2

17. A piece of string is cut in two at a random point on the string. What is the probability the longer piece is at least twice as long as the shorter piece?

(a) 1/2 (b) 1/3 (c) 3/4 (d) 2/3 (e) 1/4

18. If $f(x) = 3x + 2$, $g(x) = ax + b$ and $f(g(x)) = 15x + 8$ then $a + b =$

(a) 3 (b) 5 (c) 7 (d) 11 (e) 23

19. The coefficient of x^6y^6 in the expansion of $(x^2 + 2y)^9$ is

(a) 5376 (b) 6184 (c) 4248 (d) 2864 (e) 1982

20. If i is the square root of -1 then $(1 - i)^{10} =$

(a) 32 (b) $-32i$ (c) $10 - 10i$ (d) $16 + 16i$ (e) $64 + 32i$

21. If x ounces of a 10% solution is mixed with y ounces of a 30% solution then the result is a 15% solution. The ratio y/x then equals

(a) 1/2 (b) 3/10 (c) 1/3 (d) 2/1 (e) 3/1

22. A line with slope 3 intersects the parabola $3y = 2x^2$ at the point (3,6). The x coordinate of the other point of intersection of the line and parabola is

(a) 1/2 (b) 1 (c)5/4 (d) 3/2 (e) 7/4

23. What is the remainder of $x^{15} + 1$ divided by $x + 1$?

(a) -1 (b) 0 (c) 1 (d) 15 (e) 14

24. If x, $2x + 4$, $5x + 10$,... is a geometric progression for some number x then the fourth term is

(a) 45 (b) 75 (c) 125 (d) 175 (e) 250

25. If S is a square inscribed in a 3,4,5 right triangle and an angle of S is the right angle of the triangle then the sides of S have length

(a) 7/3 (b) 5/3 (c) 7/4 (d) 12/5 (e) 12/7

26. If N is the smallest integer such that $630N$ is a perfect square then the sum of the digits of N is

(a) 2 (b) 5 (c) 6 (d) 7 (e) 9

27. Given that the four equations

$$u + x + y + z = 5$$
$$u - x + y + z = 1$$
$$u + x - y - z = 1$$
$$u - x - y + z = 3$$

have a unique solution then $u - x + y - z =$

(a) -5 (b) -3 (c) -1 (d) 1 (e) 3

28. Given that x and y are integers, there are how many solutions (x,y) of the equation $3x + 2y = 26$?

(a) none (b) 1 (c) 2 (d) more than 2 but finitely many
(e) infinitely many

29. From the integers {0,1,2,3,4,5,6,7,8,9} John and Mary each select three numbers at random. What is the probability that Mary selects exactly two of the three numbers that John selects?

(a) 2/15 (b) 3/22 (c) 9/50 (d) 4/27 (e) 7/40

30. For which values of a is the graph of $x^2 - y^2 + 2x + a = 0$ a hyperbola?

(a) all a (b) $a > 0$ (c) $a < 1$ (d) $a \neq 1$ (e) $a \neq 0$