David Essner Exam 20 1999-2000

1. If a basketball team scores an average of 60 points in their first 11 games and 70 points in their last 19 games then the average score for all 30 games is nearest

(a) 64 (b) 65 (c) 68 (d) 67 (e) 66

2. Arcsin x could not be which of the following values?

(a) 2/3 (b) $-\pi/5$ (c) $3\pi/4$ (d) $\sqrt{2}$ (e) $1/\pi$

3. A clock has hour, minute and second hands. If the second hand is 3 inches long then through how many inches does the tip of the second hand go in 5 minutes?

(a) 480 (b) 90 (c) 15π (d) 30π (e) 300π

4. If a ball team wins 60% of their first 30 games, what percent of the remaining 60 games must they win in order to have won exactly 50% of all 90 games?

(a) 40 (b) 42 (c) 45 (d) 48 (e) 52

5. The sum of all solutions of $|x| + 2x = 1$ is

(a) 4/3 (b) 3/2 (c) 3 (d) 1/2 (e) 1/3

6. The negation of the statement '$x = 7$ or $y > 3$' is equivalent to

(a) $x \neq 7$ or $y \leq 3$ (b) $x = 7$ and $y > 3$ (c) $x \neq 7$ and $y \leq 3$ (d) if $x = 7$ then $y \leq 3$
(e) if $y \leq 3$ then $x \neq 7$

7. If x is a number very close to 1, but not equal to 1, then $\dfrac{x^3-1}{x-1}$ is

(a) near 0 (b) near 1 (c) near 2 (d) near 3 (e) a very large number

8. If x,y,z are integers each greater than 3 and are relatively prime (no common divisor greater than 1) and $x^2 + y^2 = z^2$ then the smallest possible value of $x + y + z$ is

(a) 24 (b) 30 (c) 33 (d) 37 (e) 45

9. The two roots of $x^2 + x + 1$ are also roots of $2x^3 - x^2 - x - 3$; what is the third root of the latter?

(a) 3/2 (b) 2/3 (c) -3/2 (d) - 2/3 (e) -6

10. Let the operation * on the set of real numbers be defined by $a*b = \dfrac{a+b}{1-ab}$. The inverse of 3 with respect to * is

(a) 1/3 (b) -3 (c) -1/2 (d) 0 (e)

11. Starting with x pounds of a salt solution which is 99% water, after evaporation the solution is 98% water; what is then the weight of the solution?

(a) $98x/99$ (b) $99x/100$ (c) x (d) $x - 1$ (e) $x/2$

12. Four persons A,B,C,D sit in clockwise order at a round table and each is assigned a number. If A is assigned 1 and each of A,B,C,D is assigned the average of the numbers of the two adjacent persons then B is assigned the number

(a) 1 (b) 1/2 (c) it could be any number (d) no number is possible

(e) not enough information to solve the problem.

13. If $x_0 = 1$, $x_1 = 4$ and $x_n = \frac{x_{n-1} + x_{n+1}}{2}$ for n > 0 then x_{100} =

(a) 198 (b) 98 (c) 290 (d) 360 (e) 301

14. One red, two black and three white balls are placed randomly in 3 boxes, one box having three balls, one having two balls and one having one ball. What is the probability no box has two balls of the same color?

(a) 1/6 (b) 1/5 (c) 2/9 (d) 2/11 (e) 6/35

15. Tom and Bill run a 100 yard race at constant speed and Tom wins by 10 yards. If they run the same course again, each at his same constant speed, except that Tom starts x yards behind the starting line, then they will tie at the finish if x =

(a) 10 (b) 110/9 (c) 100/9 (d) 9 (e) 91/9

16. The coefficient of $a^3b^2c^3$ in the expansion of $(a + 2b + c)^8$ is

(a) 32 (b) 386 (c) 524 (d) 2240 (e) 4260

17. If a,b are the roots of $x^2 - 6x + 1 = 0$ then $a^2 + b^2$ =

(a) 37 (b) 34 (c) 28 (d) 24 (e) 13

18. Given the point $P = (1,1)$ and circle $x^2 + y^2 - 6x - 4y + 12 = 0$ the distance d from P to the intersection point Q of the circle and the tangent line from P to the circle is

(a) 3/2 (b) $2\sqrt{2}$ (c) 2 (d) $\sqrt{3}$ (e) $\sqrt{5}$

19. If $z = 1 - \sqrt{3}\,i$, where $i^2 = -1$, then $\sqrt{z} = a + bi$ where $a + b$ =

(a) $\frac{1-\sqrt{3}}{2}$ (b) $\sqrt{3} + \sqrt{2}$ (c) $\frac{\sqrt{3}-\sqrt{2}}{2}$ (d) $\frac{\sqrt{2}-\sqrt{6}}{2}$ (e) $\frac{\sqrt{6}-\sqrt{3}}{4}$

20. For which values of x is $(x + 1)/|x - 1| > 1$?

(a) $x > 0$ (b) $0 < x < 1$ (c) $x > 1$ (d) $x > -1$, $x \neq 1$ (e) $x > 0$, $x \neq 1$

21. The equation 2 log x = log 2x (any positive base) has how many real number solutions?

(a) 0 (b) 1 (c) 2 (d) 3 (e) more than 3

22. In a circle of radius r a line drawn from the center is perpendicular to a chord of length 4. If the line intersects the chord at point P and intersects the circle at a point Q which is 1 unit from P then r =

(a) 5/2 (b) 7/2 (c) 3 (d) 9/4 (e) $4\pi/3$

23. A man invests \$$P$ in the stock market. Each of the first two years the investment increases by 20% and each of the next two years it decreases by 10%. At the end of the 4 years the value of the investment is nearest what multiple of P?

(a) 1.2 (b) 1.17 (c) 1.24 (d) 1.3 (e) 1.1

24. The numbers 8,*b*,*c* form an increasing arithmetic sequence and the numbers 9,*b*,*c* form a geometric sequence. Then the sum of the digits of *b* is

(a) 3 (b) 6 (c) 8 (d) 10 (e) 7

25. Which expression is a factor of $x^2 - xz + yz - y^2$?

(a) $x + y$ (b) $y - z$ (c) $x + y + z$ (d) $x + y - z$ (e) $x - y - z$

26. If *a*,*b*,*c* are integers then $a \equiv b \bmod c$ is defined to mean that *c* is a divisor of *b* - a . If $x \equiv y \bmod 2$ and $y \equiv z \bmod 3$ then which of the following must be true:

(*I*) $x \equiv z \bmod 6$ (*II*) $x \equiv z \bmod 3$ (*III*) $x \equiv z \bmod 8$?

(a) *I* only (b) *II* and *III* only (c) *I* and *III* only (d) all of I,*II*,*III* (e) none of *I*,*II*,*III*

27. If $A = 10^{20}$, $B = 20^{10}$, and $C = 15^{15}$ then

(a) $A > B > C$ (b) $B > A > C$ (c) $A > C > B$ (d) $B > C > A$ (e) $C > A > B$

28. How many digits are required to write all of the numbers 1,2,3,...,1000?

(a) 2664 (b) 2768 (c) 3030 (d) 2962 (e) 2893

29. A rectangle with sides of length x and y is a **golden rectangle** if $(y + x)/y = y/x$. In this case $y/x =$

(a) $1 + \sqrt{3}$ (b) 3/2 (c) $2\sqrt{2} - 1$ (d) $\frac{1+\sqrt{5}}{2}$ (e) $\frac{\sqrt{3}+\sqrt{2}}{2}$

30. For how many positive integer values of x is $\sqrt{40-\sqrt{x}}$ a positive integer?

(a) 6 (b) 1 (c) 3 (d) none (e) more than 6