David Essner Exam 21   2000-2001

1. If * is the operation on positive real numbers defined by $a*b = \frac{a}{a+b}$ then $2*(2*(2*2)) =$

   (a) 5/7   (b) 16   (c) 1/8   (d) 1   (e) 16/3

2. Given a circle with a circumscribed square with area A and an inscribed square with area a then $\frac{A}{a} =$

   (a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\pi/2$ (d) 2   (e) 4

3. If $x^2 - 3x + 2 < 0$ and $p = x^2 + 3x + 2$ then

   (a) $0 < p < 6$ (b) $3 < p < 9$ (c) $9 < p < 12$ (d) $6 < p < 12$
   (e) p can be any number

4. If the parabola $y = ax^2 + b$ passes through the points (1,1) and (2,7) then the vertex is at the point

   (a) (1,0)   (b) (0,1)   (c) (-1,0)   (d) (0,-1)   (e) (0,0)

5. A number is removed from a set of N **positive** integers. If the average of the set before removal was 90 and after removal was 100, what is the largest possible value of N?

   (a) 9   (b) 12   (c) 15   (d) 20   (e) 27

6. A drawer has 4 brown and 4 blue socks; a man randomly selects 2 socks from the drawer. What is the probability they are of the same color?

   (a) 1/2   (b) 4/7   (c) 3/8   (d) 2/3   (e) 3/7

7. Container A has x ($x > 4$) gallons of water and container B has x gallons of juice. Four gallons are removed from each container and then placed in the other; this step is then repeated and containers A and B now have the same amount of juice. What is the value of x?

   (a) 8   (b) 6   (c) 16   (d) 12   (e) 10

8. If three numbers are in increasing geometric sequence, the sum of the three numbers being 52 and the difference between the first and third being 32, then the second number is

   (a) 8   (b) 12   (c) 16   (d) 20   (e) 24

9. The price of an item is increased by x percent. By what percent of the new price must it then be decreased to bring it back to the original price?

   (a) x (b) $\frac{100+x}{100x}$ (c) $\frac{99x}{100}$ (d) $\frac{100x}{99}$ (e) $\frac{100x}{100+x}$

10. The area of the region in the Cartesian plane described by $|y - x| \leq 1$ and $|y + x| \leq 1$ is

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) 1/4 (e) 4

11. If x,y are real numbers and $x^2 + 19 - y - 8x = 0$ then

(a) $x \leq 8/19$ (b) $x \geq 3/2$ (c) $y \geq 3$ (d) $y \leq 19/8$ (e) none of (a)-(d)

12. The sum of the digits of the least common multiple of the three numbers 10,800, 45,000 and 40,500 is

(a) 6 (b) 12 (c) 11 (d) 9 (e) 14

13. Given a parallelogram with adjacent sides 5 and 10 and acute angle 60° between adjacent sides, the area of the parallelogram is

(a) 25 (b) $\frac{50}{\sqrt{3}}$ (c) $25\sqrt{2}$ (d) $50\sqrt{2}$ (e) $25\sqrt{3}$

14. A coin is to be tossed until 2 heads are obtained; the probability that at least 5 tosses is required is

(a) 5/16 (b) 5/32 (c) 1/8 (d) 3/16 (e) 2/5

15. If the squares of two consecutive integers differ by 1999 then the sum of the digits of the larger integer is

(a) 1 (b) 11 (c) 18 (d) 24 (e) 29

16. Car *1* goes from town *A* to town *B* in 4 hours and Car *2* goes from town *B* to town *A* in 6 hours. If the cars start at the same time, travel on the same road, and each goes at a constant rate, how many hours after they start do they meet?

(a) 2 (b) 12/5 (c) 9/4 (d) 10/3 (e) 20/9

17. The number of positive integer pairs (x,y) that are solutions of $2x + 3y = 97$ is

(a) 11 (b) 13 (c) 16 (d) 23 (e) 32

18. If the integer 2000_{10} (2000 to the base 10) equals the integer $abcd_8$ (written to the base 8 where a,b,c,d are integers 0,1,2...,7) then $a + b + c + d =$

(a) 10 (b) 12 (c) 13 (d) 14 (e) 16

19. The value of $[\log_{10}(2 \log_{10}100{,}000)]^3$ is

(a) 10,000 (b) 100 (c) $\log_{10}8{,}000$ (d) 1 (e) $\log_{10}9$

20. An amount of money is invested at a fixed rate of interest compounded annually. If the value of the investment doubles after 10 years then after 6 years the value is multiplied by

(a) 5/3 (b) 8/5 (c) $\log_2(\frac{25}{9})$ (d) $\log_6 10$ (e) $2^{3/5}$

21. The value of $\sqrt{4.004}$ is nearest which number?

(a) 2.002 (b) 2.001 (c) 2.0002 (d) 2.02 (e) 2.004

22. At a picnic half of the children drank coke, 1/3 of them drank pepsi, 1/3 of the difference between the coke and pepsi drinkers drank sprite, and the remaining 6 children drank something else. How many children were there?

(a) 42 (b) 48 (c) 54 (d) 60 (e) 66

23. If a,b,c are integers then $a \equiv b$ mod c is defined to mean that c is a divisor of $a - b$. If $x \equiv y$ mod 2 and $y \equiv z$ mod 3 then from the following

(I) $x \equiv z$ mod 6 (II) $x \equiv z$ mod 3 (III) $x \equiv z$ mod 5

which of these must be true?

(a) *I* only (b) *II* and *III* only (c) *I* and *III* only (d) all of *I, II,* and *III*
(e) none of *I, II, III*

24 There are how many zeros at the end of 100! (= 1x2x3x4x… x100)?

(a) 16 (b) 21 (c) 24 (d) 30 (e) 36

25. If x is a complex number such that $x^2 = x - 1$ then $x^3 =$

(a) $\frac{\sqrt{3}}{2}\,i$ (b) $1 + \frac{\sqrt{3}}{2}i$ (c) $1 - i$ (d) $\frac{\sqrt{3}}{2} - i$ (e) -1

26. If $0 < |x - 1| < 1/10$ then $y = \frac{1}{x-1} - \frac{2}{x^2-1}$ must satisfy

(a) $y > 10$ (b) $y < -10$ (c) $-1/10 < y < 1/10$ (d) $10/21 < y < 10/19$
(e) none of (a)-(d)

27. If the polynomial $P(x) = x^3 - 4x^2 + Ax + 30$ has $x = 2$ as one root then the difference between the largest and smallest root of $P(x)$ is

(a) 2 (b) 4 (c) 6 (d) 8 (e) 10

28. If $x_1 = 2$, $x_2 = 1$ and $x_n = x_{n-1} + x_{n+1}$ then $x_{100} =$

(a) 1 (b) 2 (c) -2 (d) -1 (e) 3

29. There are how many positive integers N between 1 and 1,000 inclusive such that N is divisible by at least one of the numbers 6 and 8?

(a) 320 (b) 224 (c) 168 (d) 421 (e) 250

30. A gambler, starting with an initial amount A, makes a series of 10 bets. The first bet is $A/2$, the second bet is $A/6$, the third bet is /12, and continuing the bets follow the pattern $\frac{A}{n(n+1)}$ for the nth bet. If he loses all 10 bets then the amount remaining is what fraction of A?

(a) 1/11 (b) 1/110 (c) 1/90 (d) 1/10! (e) 0