David Essner Exam 21   2001-2002

1. If $x > 0$, $y > 0$ and x^2 is 10% of x and y^3 is 9% of y then $x + y =$
    (a) 0.19   (b) 0.43   (c) 1.9   (d) 0.4   (e) 0.36

2. A basketball player averaged 6 points per game for the first 5 games, 8 points per game for the next 10 games, and then 12 points per game for the last 15 games. His average for the 30 games was then how many points per game?
    (a) 9   (b) 26/3   (c) 29/3   (d) 19/2   (e) 21/2

3. An isosceles triangle with two angles of 75° and two sides of length 10 has area
    (a) 25   (b) $15\sqrt{3}$ (c) 20   (d) $20\sqrt{2}$ (e) $10\sqrt{5}$

4. At Center High School 1/3 of the boys who play football also play baseball and 2/5 of the boys who play baseball also play football. If F boys play football and B boys play baseball then $F/B =$
    (a) 2/15   (b) 6/5   (c) 5/6   (d) 15/2   (e) 11/15

5. The sum of the first n terms of a sequence is $2n^2 + n$; the fourth term is then
    (a) 3   (b) 6   (c) 9   (d) 12   (e) 15

6. A coin is to be tossed until 2 heads are obtained; the probability that exactly 5 tosses are required is
    (a) 1/8   (b) 5/32   (c) 3/16   (d) 1/5   (e) 1/10

7. If $a \neq 0$ then the straight line $y = 2x - a$ and the hyperbola $x^2 - y^2 = a$ have exactly one point of intersection if $a =$
    (a) -2   (b) -1   (c) 1   (d) 2   (e) 3

8. Tom runs 1 mile at a constant speed of 10 miles per hour. John runs the first half-mile at a constant speed of 9 miles per hour. At what constant speed in miles per hour must John run the second half-mile in order to catch Tom at the end of 1 mile?
    (a) 45/4   (b) 23/2   (c) 11   (d) 34/3   (e) 89/8

9. If $\log_{10}2$ is approximately 0.301 then which is the best approximation for 5^{20} ?
    (a) 10^7 (b) 10^{10} (c) 10^{14} (d) 10^{16} (e) 10^{19}

10. Which is the largest number of the following?
    (a) 4^{4^2} (b) 8^{3^2} (c) 16^{2^3} (d) 2^{8^2} (e) 2^{3^4}

11. If it is assumed that each month is equally likely as a birth month , what is the probability that 3 persons chosen at random were all born in different months?
    (a) 55/72   (b) 1/2   (c) 25/36   (d) 3/4   (e) 27/48

12. A car goes 20 miles in the direction 30° north of east and then 20 miles in the direction 60° north of east. How many miles is the car from the original starting point.?

     (a) $10(\sqrt{2}+\sqrt{3})$ (b) $10(\sqrt{3}+\sqrt{6})$ (c) $20\sqrt{6}$ (d) $10(\sqrt{2}+\sqrt{6})$ (e) $20\sqrt{3}$

13. The largest prime divisor of $3^9 - 27$ is

     (a) 3   (b) 17   (c) 11   (d) 7   (e) 13

14. Glass A has 6 ounces of water and glass B has 6 ounces of acid. If one ounce is removed from A and put into B, and then one ounce is removed from B and put into A then the ratio of acid in A to that in B is

     (a) 1 to 5   (b) 1 to 6   (c) 1 to 7   (d) 1 to 12   (e) 1 to 18

15. If for some real numbers a,b the equation $bx^3 + (a + b)x^2 - 7x - 10 = 0$ has solutions $x = -1$, $x = 2$ then another solution of the equation is $x =$

     (a) -7   (b) 1   (c) 3   (d) -5   (e) 7

16. If a and b are positive integers, $a \le b$, for how many pairs (a,b) is $1/a + 1/b = 1/4$?

     (a) none   (b) 1   (c) 2   (d) 3   (e) more than 3

17. Eight players play a round robin chess tournament (each person plays each other person exactly once). Each winner earns 5 points, each loser earns no points, and in case of a tie each player earns 1 point. If there are 11 ties then the total number of points earned by all players is

     (a) 99   (b) 107   (c) 113   (d) 119   (e) 123

18. The area of the triangle bounded by the lines $y = x$, $y = 4 - x$ and $y = 4 + 2x$ is

     (a) $8\sqrt{2}$ (b) $9\sqrt{2}$ (c) 23/2   (d) 12   (e) 9

19. If a,b are positive integers and $a \le b$, then by $a \equiv b$ mod c is meant the remainder of the division of b by c is a. If $x \equiv 3^{100}$ mod 5 then $x =$

     (a) 0   (b) 1   (c) 2   (d) 3   (e) 4

20. The value of $\sqrt{17} - \sqrt{15}$ is nearest

     (a) 0.04   (b) 0.20   (c) 0.25   (d) 0.40   (e) 0.50

21. At a party there were 50 persons, including both men and women. The first woman to arrive knew 11 of the men, the next one knew 12 of the men, the next knew 13, etc and the last one to arrive knew all the men. How many women were at the party?

     (a) 16   (b) 20   (c) 24   (d) 26   (e) 29

22. If $x > 0$ then $\log_{10}(x^2 - 3)$ is a negative real number for which real number values for x?

     (a) $1 < x < \sqrt{3}$ (b) $1 < x < 3$ (c) $1/3 < x < 2$ (d) $\sqrt{3} < x < 2$ (e) $0 < x < 2$

23. The shortest distance from a point P, lying outside of a circle C of radius 5, to a point on C is 8. If a tangent line is drawn from P to C, and Q is the point of tangency then the length of segment PQ is

        (a) $5\sqrt{5}$ (b) 18   (c) $\sqrt{194}$ (d) 15   (e) 12

24. If $x(1) = \sqrt{2}$ and $x(n+1) = \sqrt{2x(n)}$ for $n = 1,2,3,...$ then $x(4) = 2^k$ where $k =$

        (a) 1/16   (b) 15/16   (c) 16   (d) 15   (e) 7/16

25. If x,y,z are real numbers and (i) $xyz = 10$ (ii) $1/x + 1/y + 1/z = 5$ and (iii) $x + y + z = 20$ then $x^2 + y^2 + z^2 =$

        (a) 160   (b) 210   (c) 300   (d) 420   (e) 540

26. Two circles of radius r each pass through the center of the other. The area of the region R lying inside both of the circles is the product of r^2 and

        (a) $2\pi/3 - \sqrt{3}/2$ (b) $2\sqrt{3} - \pi/4$ (c) $\pi/3 + \sqrt{3}/4$ (d) $\pi/4 + \sqrt{3}/2$

        (e) $\pi/2 + \sqrt{3}/4$

27. A sum S of money is invested at a fixed rate of interest r compounded annually; if the value doubles after 10 years then in how many years will it have tripled?

        (a) 15   (b) $10 \log_{10}\dfrac{3}{2}$ (c) $10^{3/2}$ (d) $10\,\dfrac{\log_{10} 3}{\log_{10} 2}$

        (e) depends on the values of r, S

28. For what value of c is there a point which lies on all three of the lines $y = 2x - 1$, $y = cx - 4$ and $2y = cx + 1$?

        (a) 1   (b) 3   (c) 4   (d) 6   (e) –1

29. If $x, 0 < x < 1,$ is a number near 1 then $\dfrac{1/x - x}{1 - \sqrt{x}}$ is

        (a) near 0   (b) near 1   (c) near 4   (d) a very large positive number

        (e) undefined

30. An army unit has S soldiers. The commander tried to place them in a square, but had 100 soldiers left over; he then tried to increase each side of the square by 1 but was lacking 61 soldiers. The sum of the digits of S is

        (a) 18   (b) 21   (c) 15   (d) 8   (e) 11