David Essner Exam 22   2002-2003

1. If one class of 30 students averaged 80 on an exam, a second class of 40 students averaged 60 on the exam and a third class of 20 students averaged 50 on the exam then the combined average of all 3 classes is nearest the integer

(a) 62 (b) 63 (c) 64 (d) 65 (e) 66

2. Given that $a*b = \dfrac{a}{a+b}$ then $x*(x*x)$ equals

(a) $\dfrac{2x}{2x+1}$ (b) $\dfrac{2x}{x+1}$ (c) $\dfrac{x}{x+1}$ (d) $\dfrac{x}{2x+1}$ (e) $\dfrac{2x}{x+2}$

3. Given that the values b,c are among the set of integers {1,2,3,4,5,6} then there are how many equations of the form $x^2 + bx + c = 0$ such that all roots are real and rational?.

(a) 4 (b) 7 (c) 8 (d) 10 (e) 12

4. In the game of basketball John made 90% of his free throws and Bill made 80% of his free throws. If they shot the same number of free throws and John missed x free throws then Bill missed how many free throws?

(a) 9x/8 (b) $8x/9$ (c) $10x$ (d) $2x$ (e) $x + 10$

5. A person invests $1,000 at a fixed rate of interest compounded 4 times per year. If after 5 years the value of the investment is $1,500, then after 10 years the value of the investment is

(a) $2,000 (b) $2,125 (c)$2,250 (d) $2,375

(e) cannot be determined from the given information

6. Each year one of the three schools Central, Western and Northeastern is equally likely to be selected to host a math competition. What is the probability that over a three year period each of the three schools is selected exactly once?

(a) 1/3 (b) 4/27 (c) 2/9 (d) 5/27 (e) 1/2

7. Given a triangle whose sides are of length 3,4,5, if h is the length of the altitude to the longest side then h equals

(a) 5/2 (b) 8/3 (c) 9/4 (d) 7/3 (e) 12/5

8. For how many positive integers n are n, $n + 2$ and $n + 4$ all prime numbers?

(a) none (b) 1 (c) 2 (d) more than 2 but a finite number

(e) an infinite number

9. If $P(x) = ax^3 + bx^2 + cx + d$ is a real number polynomial function, $P(1) = P(2) = P(-1) = 0$ and $P(-2) = -24$ then $P(3) =$

(a) 16 (b) 12 (c) 8 (d) 48 (e) 72

10. For each non-empty subset T of {1,2,3,4,5} let S_T be the sum of all numbers in T. The sum of all S_T is

(a) 180 (b) 190 (c) 200 (d) 220 (e) 240

11 Between years 1990 and 2000 at a certain university the number of boys increased by 10%, the number of girls by 40% and the total number of students by 30%. The ratio of boys to girls in 1990 was

(a) 2 to 3 (b) 1 to 3 (c) 1 to 4 (d) 3 to 4 (e) 1 to 2

12. For which values of x does the parabola $y = 5x^2 + x - 3$ lie above the parabola $y = 2x^2 + 6x - 1$?

(a) $x < -1/2$ or $x > 7/2$ (b) $x < -1/4$ or $x > 5/2$ (c) $x < -2/3$ or $x > 3$
(d) $x < -1/3$ or $x > 2$ (e) $x < -3/5$ or $x > 7/3$

13. If n is the smallest integer such that $616n$ is a perfect square, then the sum of the digits of n is

(a) 8 (b) 10 (c) 13 (d) 17 (e) 25

14. Measured by weight a given salt solution of 100 pounds is 90% water. If after evaporation the solution is by weight 60% water then the weight of the remaining solution in pounds is

(a) 70 (b) 54 (c) 40 (d) 36 (e) 25

15. If m,n are positive integers and $m + n\sqrt{2} = \sqrt{41+24\sqrt{2}}$ then $m + n =$

(a) 3 (b) 5 (c) 7 (d) 9 (e) 11

16. There are how many different (non-congruent) triangles with sides of integer length and perimeter 16?

(a) 3 (b) 4 (c) 5 (d) 6 (e) 7

17. A system of equations $ax + by = c$ and $dx + ey = f$ has solution $x = 2$, $y = 1$ when $c = 6$ and $f = 8$, and has solution $x = 1$, $y = 2$ when $c = 6$ and $f = 4$. If $c = 8$ and $f = 12$ then $x + y$ equals

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

18. Tom drives from town A to town B in 6 hours and Bill drives from town B to town A in 8 hours. If they both start at the same time and drive at a constant rate , then what is the number of hours after the starting time until they meet?

(a) 7/2 (b) 10/3 (c) 16/5 (d) 19/5 (e) 24/7

19. If a,b,c are positive real numbers and $\log_4 a = \log_6 b = \log_9(a + b)$ then $b/a =$

(a) 3/2 (b) 2/3 (c) $(\sqrt{3} - 1)/2$ (d) $(1 + \sqrt{5})/2$ (e) $\sqrt{15}/2$

20 . Circle C_1 has radius 2 and Circle C_2 has radius 3, and the distance between the centers of C_1 and C_2 is 7. If two lines, one tangent to both circles and the other passing through the center of both circles, intersect at a point P which lies <u>between</u> the centers of C_1 and C_2, then the distance between P and the center of C_1 is

(a) 9/4 (b) 7/3 (c) 8/3 (d) 13/5 (e) 5/2

21. In a league of 8 teams each team played each other team 10 times. The number of wins of the 8 teams formed an arithmetic sequence. What is the least possible number of games won by the champion?

(a) 42 (b) 45 (c) 48 (d) 50 (e) 54

22. In the coordinate plane the point $(a,0)$ has distance 2 from the line $y = 2x$; if $a > o$ then a equals

(a) 5/2 (b) 7/2 (c) $\sqrt{6}$ (d) $2\sqrt{2}$ (e) $\sqrt{5}$

23. For what value of r is the line through the points (2,0) and (0,4) tangent to the circle $x^2 + y^2 = r^2$?

(a) 2 (b) 5/2 (c) $4/\sqrt{5}$ (d) $1 + \sqrt{5}$ (e) $\sqrt{7}/2$

24. Given that $A = 2^{5/8}$, $B = 3^{1/3}$ and $C = 4^{1/4}$ then
(a) $A > B > C$ (b) $A > C > B$ (c) $C > B > A$ (d) $C > A > B$ (e) $B > A > C$

25. If $0 < x < .01$ then $\dfrac{2^{2x} - 1}{2^{x+1} - 2}$ is
(a) between 0 and 1 (b) between 1 and 2 (c) between 2 and 1,000
(d) greater than 1000 (e) less than 0

26. Let A,B,C be vertices of an equilateral triangle, and let D,E be points on the side AB such that segments AD, DE, and EB each have length 1. Then tan $\angle CDE$ equals
(a) 3 (b) $3\sqrt{2}$ (c) $2\sqrt{3}$ (d) $3\sqrt{3}$ (e) $3\sqrt{3}/2$

27. By $a \equiv b \bmod c$ is meant that (b - a) is divisible by c. If
$41 \equiv n \bmod 72$ and $k \equiv n \bmod 18$, where $0 \le k < 18$, then k equals

(a) 13 (b) 11 (c) 9 (d) 8 (e) 5

28. If 7^{100} is divided by 100 then the remainder is

(a) 1 (b) 7 (c) 14 (d) 43 (e) 49

29. Given a regular decagon (10 sided polygon), there are how many diagonals (lines joining vertices and lying inside the decagon)?

(a) 30 (b) 35 (c) 40 (d) 45 (e) 90

30. If m,n are integers and $2m - n = 5$ then $m - 3n$

(a) can be any integer (b) is a multiple of 3 (c) is an even integer
(d) is a multiple of 5 (e) is none of (a)-(d)