David Essner Exam 23 2003-2004

1. A basketball player made 75% of his first 60 free throws. After he then made his next N free throws, he had made 80% of all of his free throws. The sum of the digits of N equals

(a) 6 (b) 7 (c) 9 (d) 10 (e) 12

2. An equilateral triangle and a square have equal perimeters. The ratio of the area of the triangle to the area of the square is

(a) 3/4 (b) $\sqrt{3}/2$ (c) $3\sqrt{3}/8$ (d) $4\sqrt{3}/9$ (e) $7\sqrt{3}/12$

3. If 3/7 is written as a decimal $0.a_1a_2\,a_3\,a_4\ldots$ then a_{50} =

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

4. If $y = \log_{10}x$ and the value of x is multiplied by 100 then the value of y is

(a) multiplied by 2 (b) multiplied by 10 (c) multiplied by 100
(d) increased by 2 (e) increased by 100

5. Six seats are placed about a round table; if John and Mary are given random seating assignments then what is the probability they sit next to each other?

(a) 1/3 (b) 3/5 (c) 2/5 (d) 1/2 (e) 1/6

6. The shortest distance between a point on the circle $x^2 + y^2 = 8$ and a point on the circle $(x - 10)^2 + (y - 10)^2 = 32$ is

(a) $6\sqrt{2}$ (b) $10 - 4\sqrt{2}$ (c) $5 + 2\sqrt{2}$ (d) $2 + 6\sqrt{2}$ (e) $4\sqrt{2}$

7. In a certain school 3/4 of the boys were taking science, 3/5 of the girls were taking science, and 2/3 of all the students, boys and girls, were taking science. What is the ratio of boys to girls?

(a) 4/5 (b) 5/3 (c) 4/3 (d) 3/2 (e) 6/5

8. If 2 and 5 are the first two of a sequence of 20 numbers whose values form an arithmetic progression, then the average of the 20 numbers is

(a) 18 (b) 22.5 (c) 30.5 (d) 28.5 (e) 26

9. For which value(s) of a does the system of three equations

$ax - 2y + z = 0$; $x - y + 2z = 1$; $2x + y - z = 0$

not have a solution for x,y,z?

(a) $a = 5$ (b) $a = -7$ (c) $a = 2/3$ (d) there are many such values for a
(e) there are no such values for a

10. A circle has radius 1 and center O. If the distance from O to a chord C of the circle is 1/2 then what is the area of the region which lies between C and the circle and does not include O?

(a) $(\pi - \sqrt{3})/2$ (b) $\pi/2 - \sqrt{3}$ (c) $(\pi - \sqrt{2})/3$ (d) $(\pi - 2)/\sqrt{3}$ (e) $\pi/3 - \sqrt{3}/4$

11. If x - 1 is a factor of $5x^4 - bx^3 + 7x^2 + 3x - 11$ then b =
(a) 14 (b) 7/55 (c) - 21 (d) 15 (e) 4

12. If the real number x, $x > 0$, satisfies the equation $x + 1/x = 3$ then $x^4 + 1/x^4$ equals
(a) 81 (b) 81 1/81 (c) 47 (d) 16 (e) 63

13. A container has 10 ounces of a 40% (by weight) sodium solution. How many ounces of a 70% (by weight) sodium solution must be added to the container to yield a 50% solution?
(a) 3.5 (b) 4.2 (c) 4.7 (d) 5 (e) 5.6

14. The number 25! (= 1x2x3x4x…x25) has how many different prime factors?
(a) 5 (b) 9 (c) 15 (d) 39 (e) 120

15. At what rate of interest does an investment double in 10 years if the interest is compounded 4 times per year?
(a) $4(2^{1/40} - 1)$ (b) $(2^{1/10})/40$ (c) $4(1 + 2^{1/10})$ (c) $4/(1 + 2^{1/40})$ (e) $1 + 2^{1/10}/4$

16. Let A,B,C be vertices of a triangle such that $AB = x$, $AC = y$ and $\angle ABC = 60°$; then there is exactly one possible value for BC provided y/x =
(a) $\sqrt{2}/2$ (b) $2/\sqrt{3}$ (c) $3/\sqrt{2}$ (d) $\sqrt{3}/2$ (e) $\sqrt{3}/\sqrt{2}$

17. Train I goes from town A to town B and train II goes from B to A; train I travels twice as fast as II. If they start at the same time then they meet at point X. If train I starts 5 minutes later than train II then they meet at a position 2 miles from X. How fast does train I go (in miles per hour)?
(a) 50 (b) 60 (c) 64 (d) 72
(e) The answer depends on the distance between A and B

18. In the Cartesian plane (x,y) is a lattice point if x and y are both integers. How many lattice points are there on the line segment with endpoints (2,3) and (30,38) (include both endpoints)?
(a) 2 (b) 4 (c) 5 (d) 7 (e) 8

19. If $a = 2^{100}$, $b = 3^{70}$, $c = 5^{50}$ then
(a) $a < b < c$ (b) $a < c < b$ (c) $b < a < c$ (d) $b < c < a$ (e) $c < a < b$

20. If $a,b,c,x > 0$ and $\log_a x = 10$, $\log_b x = 5$ and $\log_{abc} x = 2$ then $\log_c x$ =
(a) 2 (b) 5 (c) 10 (d) 2/5 (e) 5/2

21.There are how many ordered triples (i,j,k) where i,j,k are non-negative integers (i.e. 0,1,2,3,…) and $i + j + k = 10$? (Note: (1,3,6) and (3,1,6) are different)
(a) 45 (b) 66 (c) 90 (e) 120 (f) 720

22. If the sides of a rhombus have length x and the diagonals have lengths 2 and 3 then x equals

(a) 3/2 (b) $\sqrt{3/2}$ (c) $2\sqrt{6}/3$ (d) $\sqrt{13}/2$ (e) $3/\sqrt{2}$

23. In major league baseball there are two leagues, the National League with 16 teams and the American League with 14 teams. Each team plays 162 games in a season, some games against teams in their own league and some against teams in the other league. If each team is to play the same number of games against teams in the other league, and each American League team plays x ($x > 0$) games against National League teams, what is the smallest possible value for x?

(a) 2 (b) 5 (c) 6 (d) 8 (e) 9

24. In tic-tac-toe one begins with a size 3x3 square divided into a 9 sub-square array consisting of 3 rows and 3 columns. Suppose an *X* and an *O* are each randomly placed in different sub-squares; what is the probability the two sub-squares have a side in common?

(a) 1/2 (b) 1/3 (c) 4/9 (d) 25/81 (e) 29/72

25. If $x_0 = \pi$ and $x_{n+1} = \dfrac{x_n}{(1+x_n)}$ for $n = 0,1,2,3,...$ then x_{100} equals

(a) $\pi/(100+\pi)$ (b) $\pi/(99+\pi)$ (c) $\pi/(1+100\pi)$ (d) $99/(100+\pi)$ (e) $100/(1+99\pi)$

26. For how many positive integers n is $4n + 1$ a perfect square?

(a) 1 (b) 2 (c) 3 (d) more than 3 but a finite number (e) an infinite number

27. If $a > 10$ then the sum of all solutions of $|x-5| + |x+3| = a$

(a) equals 1 for some a (b) is always 2 (c) is negative for some a
(d) is always greater than 10 (e) is always between 5 and 10

28. What is the minimum value, for all real numbers x and y, of the expression $3x^2 - 6xy + 4y^2 - 4y + 11$?

(a) -1 (b) 2 (c) 9/2 (d) 8/3 (e) 7

29. If the graph of $y = x^2 + 1$ is rotated 90° clockwise about the origin, then shifted 2 units to the right and then 2 units up, the resulting graph contains the point

(a) (7,4) (b) (9,4) (c) (8,3) (d) (8,4) (e) (9,3)

30. The statement "$x < 1$ and (if $y < 7$ then $x > 3$)" implies

(a) $x + y > 2$ (b) $xy > 7$ (c) $xy \le 7$ (d) $x - y \le 5$ (e) $y/x \ge 7$