David Essner Exam 24   2004-2005

1. The class average of 30 students on an exam was 67.2. If two students in the class scored 0 then the average of the other 28 students was
    (a) 72.4   (b) 71.6   (c) 73.2   (d) 74   (e) 72

2. The initial price of a dress was $x (where x is an integer). This price was reduced by 20% and the reduced price was reduced by another 20% to give a final price of $49.92. The sum of the digits of x equals
    (a) 10   (b) 11   (c) 13   (d) 15   (e) 17

3. If $f(x) = Ax + B$ for all real numbers x and $f(2x + 1) = x$ then B equals
    (a) -1/2   (b) 1/2   (c) 2   (d) 1   (e) 0

4. In the Cartesian plane the area of the triangle with vertices (0,0), (3,5) and (6,2) is
    (a) 10   (b) 12   (c) 27/2   (d) 21/2   (e) 40/3

5. If $a \neq 3$ and the points $(a,3)$, $(3,a)$ and $(1,4)$ are collinear then a =
    (a) 7/3   (b) 3/2   (c) 2   (d) 1   (e) 5/2

6. Box 1 has 3 red and 2 green balls and box 2 has 4 red and 1 green ball. A box is selected at random and a ball is drawn. If the ball is red, what is the probability it came from box 1?
    (a) 1/2   (b) 2/5   (c) 3/5   (d) 7/10   (e) 3/7

7. There are how many integer pairs (x,y) such that $1 \leq x < y \leq 40$?
    (a) 400   (b) 780   (c) 720   (d) 640   (e) 800

8. Initially glass A holds 10 ounces of water and glass B holds 10 ounces of wine. Then x ounces of water is transferred from A to B, mixed with the wine, and then x ounces of the mixture is transferred from B to A. If A now holds 8 ounces of water then x equals (in ounces)
    (a) 2   (b) 4/3   (c) 3   (d) 5/2   (e) 7/3

9. The integer 525 cannot be written as the sum of n consecutive integers for which value of n?
    (a) 3   (b) 4   (c) 5   (d) 6   (e) 7

10. The score of a certain multiple choice exam is computed as the number of right answers minus one fourth of the number of wrong answers. If the number of questions answered is N and the score is S then the number of right answers is
    (a) $\dfrac{4S + N}{5}$ (b) $\dfrac{5S - N}{4}$ (c) $\dfrac{3S + N}{4}$ (d) $\dfrac{4S - N}{5}$ (e) $\dfrac{4S - N}{3}$

11. Assuming both investments are compounded annually at the same rate, if the amount $100 earns $100 interest in 10 years then what amount S in dollars earns $200 interest in 20 years?

     (a) 200/3   (b) 100   (c) 75   (d) 80   (e) 50

12. Two wires connect two poles so that there is a wire from the top of each pole to the base of the other pole. If the wires cross at a distance of 40 feet above the ground and one pole is 100 feet high then the other pole is how many feet high?

     (a) 60   (b) 80   (c) 160/3   (d) 200/3   (e) 324/5

13. Given a triangle with sides of length 2,3,4, what is the value of $\sin A$ where A is the angle between the sides having lengths 2 and 3?

     (a) 1/4   (b) 3/7   (c) $\sqrt{15}/4$ (d) $\sqrt{10}/4$ (e) $4/\sqrt{29}$

14. Which of the sets of three numbers, as lengths of the sides of triangles, gives the triangle with the largest area?

     (a) 6,7,8   (b) 6,8,8   (c) 6,8,9   (d) 6,8,10   (e) 6,8,11

15. If the polynomial $P(x) = x^3 - 4x^2 + Ax + 30$ has $x = 2$ as one root then the difference between the largest and smallest root of $P(x)$ is

     (a) 2   (b) 4   (c) 10   (d) 6   (e) 8

16. Given the sequence $x_1, x_2, \ldots, x_{10}$ such that $x_1 = 1$, $x_{10} = 100$ and $x_{n+2} = x_n + x_{n+1}$ for $n = 1,2, \ldots ,8$, then $x_2 =$

     (a) 101/9   (b) 79/34   (c) 11/2   (d) 34/11   (e) 111/79

17. Tom and Jerry run a long distance race at respective rates of 11 and 9 miles per hour around an oval track which is 1/4 mile in length. If they start at the same time, how many miles after the start does Tom run before he catches Jerry for the first time?

     (a) 11/9   (b) 11/4   (c) 11/3   (d) 11/8   (e) 11/16

18. The remainder of the division of $(1! + 2! + 3! + \ldots + 17!)$ by 6 is

     (a) 1   (b) 2   (c) 3   (d) 4   (e) 5

19. The sum of all integers in the set of numbers x such that $x(x - 2)(x + 3)(x + 5) < 0$ is

     (a) – 3   (b) -6   (c) 6   (d) 0   (e) 11

20. Given the numbers $a = 2^{60}$, $b = 3^{45}$, $c = 5^{30}$ then

  (a) $a < b < c$ (b) $c < a < b$ (c) $b < a < c$ (d) $b < c < a$ (e) $a < c < b$

21. If x,y,z are respectively in the ratio 3:2:1 and $x + y + z = 1$ then the ratio of $(1 - x)$ to $(1 - y)$ is

     (a) 1:2   (b) 2:3   (c) 3:4   (d) 3:5   (e) 4:5

22. If a,b are real numbers and $a^{1/2} + b^{1/3} = 1$, $a + b^{2/3} = 5$ then a equals

     (a) 9/4   (b) 9   (c) 4/25   (d) 16/9   (e) 4

23. If $|x - 1| < .001$, $x \neq 1$ then $p(x) = (x^2 + 3x - 4)/(x^2 - x)$ must satisfy
    (a) $p(x) < 0$ (b) $0 < p(x) < 2$ (c) $2 < p(x) < 4$ (d) $4 < p(x) < 6$ (d) $p(x) > 6$

24. For what number a does the following set of three equations have more than one solution for x,y,z: $ax - 2y + 5z = 0$; $x + 2z = 1$; $2y + z = 3$?
    (a) 1   (b) 2   (c) 3   (d) 4   (e) 5

25. If ABC is a right triangle with length of side $AB = 3$ and hypotenuse $AC = 5$, then what is the length of the angle bisector AD of $\angle BAC$ where D is on the side BC?

    (a) $4\sqrt{3}/5$ (b) $5\sqrt{3}/4$ (c) $4\sqrt{5}/3$ (d) $3\sqrt{5}/2$ (e) $5\sqrt{2}/3$

26. If $1/\log 2 + 1/\log 4 = 1/\log c$ then c equals (where the log base is any number greater than 1):
    (a) 2   (b) 8   (c) 1/8   (d) $2^{1/3}$ (e) $4^{1/3}$

27. The equation $5x + 51y = 551$ has two solution pairs where x and y are positive integers. One solution pair is $(100,1)$. If (x,y) is the other solution pair then $x + y$ equals
    (a) 43   (b) 62   (c) 55   (d) 75   (e) 67

28. A and B play a series of games; the winner of each game has probability 0.8 of winning the next game. If A wins the first game, what is the probability A will win the third game?
    (a) .68   (b) .76   (c) .72   (d) .60   (e) .64

29. Two lines are each tangent to a circle of radius r, $r \geq 10$ and intersect at a right angle. If a point on the circle is at a distance of 2 from one of the lines and 9 from the other then r equals
    (a) 11   (b) 17   (c) 15   (d) 13   (e) 18

30. The statement 'it is false that if $x < 7$ then $x < 4$' is true for those values of x which satisfy
    (a) $x < 4$ or $x \geq 7$ (b) $4 \leq x < 7$ (c) $4 < x \leq 7$ (d) $x < 4$ (d) $x \leq 4$ or $x > 7$