

David Essner Exam 25 2005-2006

1. If the circumference C of a circle increases by 1 unit then the diameter increases by how much?

- (a) π (b) 2 (c) $1/\pi$ (d) $1/2$ (e) depends on the value of C

2. Given a triangle ABC if there are the lengths $|AB| = 2$, $|AC| = 3$ and the angle $\angle BAC = 60^\circ$ then $|BC|$ equals

- (a) $2\sqrt{3}$ (b) $\sqrt{10}/2$ (c) $\sqrt{5}$ (d) $2\sqrt{2}$ (e) $\sqrt{7}$

3. Let there be given a sequence of terms such that the first term is a , the second term is b and each of the other terms is the sum of the two preceding terms in the sequence. If the 6th term of the sequence is 21 and the 7th term of the sequence is 34 then the 3rd term of the sequence is

- (a) 4 (b) 5 (c) 6 (d) 7 (e) 8

4. In the coordinate plane let $P = (3, -2)$ and $Q = (x, y)$. If $(-1, 5)$ is the midpoint of the line segment PQ then $x + y$ equals

- (a) 4 (b) 5 (c) 6 (d) 7 (e) 8

5. Two balls were placed in a box; each ball was equally likely to be red or green. If it is known that at least one of the balls is red then what is the probability that both balls are red?

- (a) $1/2$ (b) $1/3$ (c) $1/4$ (d) $2/5$ (e) $2/3$

6. If b and c are integers then the sum of all the roots of the equation $x^2 + bx + c = 0$

- (a) is always an integer (b) is always a rational number but not necessarily an integer (c) is always a real number but not necessarily a rational number (d) may or may not be a real number (e) is never a real number

7. A container has 10 pounds of a 20% saline solution. How many pounds of salt must be added to the container to obtain a 40% saline solution?

- (a) 2 (b) 3 (c) $5/2$ (d) $10/3$ (e) $7/2$

8. A student got 17 correct answers on the first 20 questions of a 75 question exam and more than 90% of the answers correct on the entire test; the smallest possible number of correct answers on the last 55 questions was then

- (a) 48 (b) 49 (c) 50 (d) 51 (e) 52

9. If the integer x is 20 more than y and 10% more than y then the sum of the digits of x is

- (a) 2 (b) 4 (c) 8 (d) 10 (e) 13

10. In the Cartesian plane what is the area of the region of points (x,y) which satisfy the inequality

$$|x| + |y| \leq 1 ?$$

- (a) 1 (b) $3/2$ (c) 2 (d) $2\sqrt{2}$ (e) $3\sqrt{2}$

11. If one writes each of the integers from 1 to 1,000 inclusive, then how many digits are written?

- (a) 2460 (b) 2841 (c) 2863 (d) 2887 (e) 2893

12. If the two digits of the integer x , $11 \leq x \leq 99$, are reversed then the resulting number is 14 less than $2x$. The sum of the digits of x equals

- (a) 4 (b) 5 (c) 6 (d) 7 (e) 8

13. At a certain school 92 students take math, 56 take science and 48 take both math and science. How many students take either math or science, but not both?

- (a) 52 (b) 36 (c) 46 (d) 12 (e) 18

14. A person has \$3.60 in coins which consists of pennies, nickels and dimes, at least one of each. If there are $2/3$ as many nickels as pennies, what is the largest possible total number of coins?

- (a) 80 (b) 90 (c) 100 (d) 110 (e) 120

15. If \$ P is invested at an annual rate of r , compounded 4 times per year, then the investment will double in value in 8 years if $r =$

- (a) $1/32$ (b) $\log_2(33/32)$ (c) $2^{65/32} - 4$ (d) $2^{3/32} - 1$ (e) value depends on P

16. If $\sin x + \cos x = 5/4$ then $\sin 2x$ equals

- (a) $3/16$ (b) $5/16$ (c) $7/16$ (d) $9/16$ (e) $11/16$

17. If $\log_4 a + 3 \log_8 b = 1$ then a equals

- (a) $4/b^2$ (b) $2b^3$ (c) b^2 (d) $2/b^3$ (e) $2b^6$

18. On a math test the boys got $3/4$ of the answers correct, the girls got $2/3$ of the answers correct and the entire class got $7/10$ of the answers correct. The ratio of boys to girls is then

- (a) 3 to 4 (b) 4 to 7 (c) 3 to 5 (d) 2 to 3 (e) 7 to 20

19. There are how many positive integer pairs (m,n) which satisfy the equation $2m + 7n = 835$?

- (a) 44 (b) 51 (c) 60 (d) 71 (e) 83

20. The solution set of the inequality $|x^2 - 4x - 1| > |x^2 - 4x + 1|$ is an interval of length
- (a) 2 (b) 4 (c) 6 (d) 8 (e) 16
21. If x and y are positive integers and $x^2 - y^2 = 403$ then the smallest possible value of y is
- (a) 7 (b) 9 (c) 11 (d) 13 (e) 17
22. An equilateral triangle whose sides are of length 2 is inscribed in a circle of radius r ; then r equals
- (a) $\sqrt{2}$ (b) $(\sqrt{2} + 1)/2$ (c) $(\sqrt{3} + 1)/2$ (d) $\sqrt{3}$ (e) $2/\sqrt{3}$
23. If $a = 1110/1111$, $b = 2221/2223$, $c = 3331/3334$ then
- (a) $a > b > c$ (b) $c > a > b$ (c) $c > b > a$ (d) $b > a > c$ (e) $b > c > a$
24. What two integers between 60 and 70 divide the number $2^{24} - 1$?
- (a) 61 and 64 (b) 62 and 69 (c) 63 and 65 (d) 64 and 66 (e) 63 and 67
25. Given the three sets of numbers $A = \{1,2,3,4\}$, $B = \{5,7,8\}$ and $C = \{9,10,11\}$, the sum of all products abc , where a is in set A , b is in the set B and c is in the set C , equals
- (a) 6000 (b) 6280 (c) 6340 (d) 6446 (e) 7208
26. The graph of $y = (x^2 - 2x + 2)(x^2 + 2x - 2)(x^3 + 1)^2$ contains how many points on the x axis?
- (a) 3 (b) 4 (c) 5 (d) 6 (e) 8
27. If $3^{2x} + 3^{4x} = 20$ then 3^{3x} equals
- (a) 6 (b) 8 (c) 10 (d) $\log_3 80$ (e) $\log_2 40$
28. Bill and Tom take turns rolling a single 6 sided die until one of them rolls a 3 and is then the winner. If Bill goes first what is the probability he is the winner?
- (a) $2/3$ (b) $4/7$ (c) $19/36$ (d) $7/12$ (e) $6/11$
29. Given that four of the values (a)-(e) are solutions of the equation $x^4 - 31x^3 + 321x^2 - 1241x + 1430 = 0$, which one is not a solution?
- (a) 2 (b) 5 (c) 7 (d) 11 (e) 13
30. Tom leaves home at the same time every weekday to get his wife Mary at work and then drive her home. One day Mary got off work an hour early and began walking home until she met Tom who drove her the rest of the way. If they got home 40 minutes earlier than usual, how many minutes did Mary walk?
- (a) 20 (b) 25 (c) 30 (d) 35 (e) 40

