David Essner Exam 26 2006-2007

1. If a is 30% of b then $3a$ is what percent of $4b$?
   (a) 20.5 (b) 22.5 (c) 24 (d) 25 (e) 27.5

2. The first three terms of a sequence are 1,7,8. If the average of the first four terms is 7 then the sum of the digits of the fourth term is
   (a) 3 (b) 4 (c) 6 (d) 8 (e) 9

3. If $\frac{x}{y-6} = \frac{y}{z-3} = \frac{z}{x-4} = 2$ then x equals
   (a) 9 (b) 12 (c) 8 (d) 7 (e) 10

4. A student took 3 tests. The score on the second test was 1/3 more than the score on the first test and 1/4 more than the score on the third test. If the score on the first test was x then the score on the third test was
   (a) $4x/3$ (b) $3x/4$ (c) $7x/6$ (d) $12x/11$ (e) $16x/15$

5. For real numbers a,b,c if $\frac{a}{b} > 1$ and $ac < 0$ then necessarily
   (a) $a > b$ (b) $a + b > c$ (c) $abc > 0$ (d) $a + b + c > 0$ (e) none of (a)-(d)

6. If x,y,z are positive integers, $x \geq \frac{y}{3}$, $y \geq \frac{z}{5}$ and $y + z \geq 65$ then the smallest possible value of x is (a) 6 (b) 5 (c) 2 (d) 4 (e) 3

7. If $\sin x + \cos x = 5/4$ then $\sin 2x =$
   (a) 3/16 (b) 7/16 (c) 5/16 (d) 11/16 (e) 9/16

8. Let N denote the smallest positive integer that satisfies the two conditions:
   (i) If N is divided by 11 then the remainder is 9
   (ii) If N is divided by 7 then the remainder is 5.
   The sum of the digits of N is
   (a) 10 (b) 11 (c) 12 (d) 13 (e) 14

9. There are how many pairs of positive integers (a,b) such that $1 \leq a < b \leq 10$?
   (a) 24 (b) 36 (c) 45 (d) 64 (e) 81

10. Let $ABCD$ be a trapezoid with altitude of length 6 and bases AB of length 4 and DC of length 8. If P is the point of intersection of the diagonals AC and BD then the distance from P to DC is
    (a) 4 (b) 7/2 (c) 9/2 (d) 14/3 (e) there are many possible answers

11. Suppose in the year 1 that \$1 was invested at a rate compounded annually so that its value would double every 20 years. The value, 2000 years later, in the year 2001, would be best approximated by
    (a) \$1,300 (b) \$1,300,000 (c) $\$ 1.3\text{x}10^{20}$ (d) $\$ 1.3\text{x}10^{30}$ (e) $\$1.3\text{x}10^{72}$

12, If $x = \log_2 7$ then $7^{3/x}$ equals
    (a) 49/2 (b) 27/2 (c) 8 (d) 49/4 (e) 21/2

13. Bill and Tom start at the same place and run around an oval track in opposite directions. If they start at the same time and always run at a constant rate then they meet in one minute. If Tom starts 30 seconds before Bill then they meet 50 seconds after Bill had started. How many seconds does it take Bill to run around the track one time?
(a) 85 (b) 90 (c) 175/2 (d) 185/2 (e) 195/2

14. If the line $y = -1/2\ x + 11/2$ is tangent to the circle C at the point (3,4) and the line $y = -2x + 12$ is tangent to C at the point (5,2) then the radius of C is
(a) 9/2 (b) 14 (c) $3\sqrt{2}$ (d) $2\sqrt{5}$ (e) $2\sqrt{3}$

15. Container A has 10 pounds of water and container B has 10 pounds of a 20% salt solution. Five pounds are transferred from A to B and mixed with B, and then y pounds are transferred from B to A and mixed with A. If the final mixture in A is a 5% salt solution then y equals
(a) 3 (b) 7/2 (c) 17/6 (d) 25/9 (e) 4

16. Two balls are drawn at random, without replacement, from a bowl containing 5 red and n green balls. What is the minimum value of n such that the probability is at least 1/2 that both of the drawn balls are green?
(a) 9 (b) 11 (c) 13 (d) 15 (e) 17

17. How many (decimal) integers n, $31 < n < 64$, have more 0's than 1's in their binary representation?
(a) 6 (b) 8 (c) 10 (d) 13 (e) 16

18. The graph of $y = x^2$ is shifted one unit to the right and two units up, and then is rotated 180 degrees about the origin. A point in the resulting graph is
(a) (3,-8) (b) (2,-11) (c) (1,-5) (d) (0,-2) (e) (-1,-1)

19. If $A = 101/205$, $B = 203/409$ and $C = 299/601$ then
(a) $B > C > A$ (b) $B > A > C$ (c) $C > B > A$ (d) $A > B > C$ (e) $C > A > B$

20. Given the quadratic $p(x) = x^2 + bx + c$, if $p(3) - p(2) = 7$ then $p(4) - p(3)$ equals
(a) 9 (b) 11 (c) 14 (d) 21 (e) cannot determine from the given data.

21. Given the sequence $x_1, x_2, ..., x_{10}$, if $x_1 = 1$, $x_{10} = 100$ and $x_{n+2} = x_n + x_{n+1}$ for $n = 1,2,...,8$ then $x_2 =$
(a) 101/9 (b) 79/34 (c) 11/2 (d) 34/11 (e) 111/79

22. Team A plays team B a series of games. If in each game the probability that A wins is 2/3, then what is the probability that A wins two games before B wins two games?
(a) 8/9 (b) 9/16 (c) 20/27 (d) 64/81 (e) 7/9

23. The point $A = (3,1)$ is reflected about the x axis to give point B, and B is reflected about the line y = x to give point C = (a,b). Then b - a equals
(a) 2 (b) 3/2 (c) 3 (d) 4 (e) 9/2

24. The equation $|x + 1| = |2x - 1| + c$, where $c > 0$, has exactly one solution if c equals
(a) –3/2 (b) –1/2 (c) 1/2 (d) 3/2 (e) 7/2

25. If x is a real number and $0 < |x-2| < .1$ then the value of $f(x) = \dfrac{1/x - 1/2}{x-2}$ is in which interval?

(a) (-1,0) (b) (6, 9) (c) (3/5,7/2) (d) (7/2,6) (e) (0,3/5)

26. The equation $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{7}$ has how many solution pairs (x,y) where x and y are positive integers and $y \geq x$?

(a) 1 (b) 2 (c) 4 (d) 6 (e) more than 6

27. An equal number of boys and girls attended the senior prom. If 40% of the seniors who attended were girls and 30% of the non-seniors who attended were boys, what fraction of the girls were seniors?

(a) 2/5 (b) 7/10 (c) 5/11 (d) 8/15 (e) 3/8

28. If a right triangle has legs of length a and b and hypotenuse c then the length of the altitude to the hypotenuse equals

(a) $\dfrac{c}{\sqrt{ab}}$ (b) $\dfrac{c}{ab}$ (c) $\dfrac{\sqrt{ab}}{c}$ (d) $\dfrac{ab}{c}$ (e) $\dfrac{a+b}{\sqrt{c}}$

29. If n is a positive integer, $n > 4$, then what is the remainder if $n^{100} + 1$ is divided by $n - 1$?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

30. If $x^2 + ax + 1$ is a factor of $x^3 + 5x^2 - 13x - 2$ then $a =$

(a) 7 (b) 3 (c) -5 (d) 9 (e) 4