

David Essner Exam 27 2007-2008

1. Given three concentric circles of radius 2, r and 6 where $2 < r < 6$, if the area between the outer circle and the middle circle equals the area between the middle circle and the smaller circle then r equals

- (a) 4 (b) $4\sqrt{2}$ (c) $2\sqrt{5}$ (d) $2\sqrt{6}$ (e) 5

2. Of a group of persons who took a certain actuarial exam, $2/3$ passed the first time, $3/5$ of those who failed the first time passed the second time, and $1/2$ of those who failed the first two times passed the third time. What fraction failed all three times?

- (a) $1/15$ (b) $3/20$ (c) $4/45$ (d) $1/5$ (e) $5/64$

3. How many integers between 101 and 999 are perfect squares?

- (a) 17 (b) 21 (c) 24 (d) 27 (e) 30

4. Taxi fares increased from 20 cents per $1/11$ mile to 40 cents per $1/6$ mile. The percent increase is nearest what integer?

- (a) 5 (b) 7 (c) 9 (d) 11 (e) 13

5. In a group of students the average weight of the girls was 110 and the average weight of the boys was 160. If the average weight of all students in the group was 140 then the ratio of boys to girls was

- (a) 5 to 3 (b) 5 to 2 (c) 4 to 3 (d) 3 to 2 (e) 7 to 5

6. If $a = \log_{10}2$ then $\log_{10}125$ equals

- (a) $3 - 3a$ (b) $2 - 2a$ (c) $1/2 + 2a$ (d) $1 + a$ (e) $3a - 1/2$

7. If the parabola $y = ax^2 + bx + c$ has its vertex in the fourth quadrant and contains a point in the first quadrant then necessarily

- (a) $a > 0$ and $c > 0$ (b) $a > 0$ and $b < 0$ (c) $b < 0$ and $c > 0$
(d) $a > 0$ and $c < 0$ (e) $b > 0$ and $c < 0$

8. What is the minimum value, for all real numbers x and y , of the expression

$$3x^2 - 6xy + 4y^2 - 4y + 11?$$

- (a) -1 (b) 2 (c) $9/2$ (d) $8/3$ (e) 7

9. A baseball team averages x runs per game in the first n games. If the team then scores y runs in the next game, by how much is the average number of runs per game then changed ?

- (a) $(y - x)/(n + 1)$ (b) $(y - x)/n$ (c) $y/(n + 1) - x/n$
(d) $y/n - x/(n + 1)$ (e) $2(y - x)/((2n + 1))$

10. If $x^2 - x$ divides $p(x) = x^4 - x^3 + x^2 + ax + b$ then $a + b$ equals

- (a) -2 (b) -1 (c) 1 (d) 3 (e) 5

11. There are how many positive integer divisors of 720 (including 1 and 720)?
 (a) 18 (b) 21 (c) 24 (d) 27 (e) 30
12. An investment of \$1,000 at a rate of r percent compounded annually was worth \$1,500 after 4 years. At the same rate what would be the value of the investment after another 8 years?
 (a) \$2,500 (b) \$2,875 (c) \$3,025 (d) \$3,150 (e) \$3,375
13. In a certain city, $\frac{4}{7}$ of the men are married to $\frac{3}{5}$ of the women. What fraction of the adult population is married?
 (a) $\frac{12}{35}$ (b) $\frac{24}{35}$ (c) $\frac{41}{70}$ (d) $\frac{24}{41}$ (e) $\frac{7}{12}$
14. A bicyclist made a 50 mile trip. If he had gone an average of 5 miles per hour (mph) faster then the time of the trip would have been 30 fewer minutes. What was the average speed of the bicyclist in mph?
 (a) 15 (b) $\frac{35}{2}$ (c) $\frac{37}{2}$ (d) 20 (e) 22
15. There are how many solutions of the equation $x + y + z = 20$ where x, y, z are positive integers? (Note: $x = 1, y = 1, z = 18$ and $x = 18, y = 1, z = 1$ are different solutions.)
 (a) 156 (b) 171 (c) 185 (d) 201 (e) 224
16. If $n > 3, x > 0$ and $x^n + 2x - 1 = 0$ then
 (a) $|x - \frac{1}{4}| < 0.1$ (b) $|x - \frac{1}{2}| < 0.1$ (c) $|x| < .1$ (d) $\frac{1}{2} < x < 1$ (e) $x > 1$
17. What is the sum of all real number roots of the equation $x^2 - 2 = 2|2x + 1|$?
 (a) 0 (b) $2 + 4\sqrt{2}$ (c) $4 + \sqrt{2}$ (d) $-2 + 2\sqrt{2}$ (e) $4 - 2\sqrt{2}$
18. What is the sum of all numbers $x, 0 \leq x \leq 2\pi$, such that $\sin(2x) = \frac{\sqrt{2}}{2}$?
 (a) π (b) $\frac{3\pi}{2}$ (c) 2π (d) $\frac{5\pi}{2}$ (e) 3π
19. Which of the numbers (a)-(e) is the largest?
 (a) $\sqrt[5]{3}$ (b) $\sqrt[6]{4}$ (c) $\sqrt[8]{6}$ (d) $\sqrt[9]{7}$ (e) $\sqrt[12]{10}$
20. If x, y are positive integers and $x^2 - y^2 = 100$ then the sum of the digits of x is
 (a) 4 (b) 6 (c) 7 (d) 8 (e) 10
21. Two positive integers are *relatively prime* if their greatest common divisor is 1. How many positive integers greater than 1 and less than 30 are relative prime to 100?
 (a) 7 (b) 9 (c) 11 (d) 13 (e) 17

22. If the first term of a geometric sequence of positive numbers is 2, and the third term is 18, then the sum of the digits of the fifth term is

- (a) 9 (b) 8 (c) 6 (d) 5 (e) 11

23. In a permutation (ordering) of the sequence 1,2,3,4,5,6 an integer n is **fixed** if it is in the n^{th} position; for example 3 and 5 are fixed in the permutation 2,1,3,6,5,4. There are how many permutations of 1,2,3,4,5,6 which have exactly 3 fixed integers?

- (a) 24 (b) 30 (c) 40 (d) 60 (e) 80

24. Let A, B, C be vertices of a right triangle with $\angle C = 90^\circ$. If P is a point on AC such that $|AP| = 6$, BP bisects $\angle CBA$, and $\angle BPC = 60^\circ$ then $|BC|$ equals

- (a) 4 (b) $4\sqrt{2}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$ (e) $3\sqrt{3}$

25. If N is the number of digits in the number 2^{3000} then

- (a) $N \leq 500$ (b) $500 < N \leq 900$ (c) $900 < N \leq 1000$

- (d) $1000 < N \leq 5000$ (e) $N > 5000$

26. Let A, B, C be vertices of an equilateral triangle with sides of length 1, and P a point interior to the triangle. If a, b, c are the distances from P to the sides of the triangle then $a + b + c$ equals

- (a) $\sqrt{3}/2$ (b) $3/4$ (c) $\sqrt{2}$ (d) $2/3$ (e) Depends on the location of P

27. A circle passes through two adjacent vertices A, B of a square and is tangent to the side opposite the side having endpoints A and B . If the sides of the square have length 8 then the radius of the circle is

- (a) $19/4$ (b) $4\sqrt{2}$ (c) 5 (d) $9/2$ (e) $16/\pi$

28. Starting with a mixture of a ounces of ingredient A , b ounces of ingredient B and c ounces of water, then x ounces of A and y ounces of B are added to produce a mixture which is 25% each of A and B . Then $x + y$ equals

- (a) $a + b + c$ (b) $4a + 4b + c$ (c) $a + b - 4c$ (d) $c - a - b$

- (e) $(4a + 4b) - c$

29. Circle C is centered at $(0,2)$ and has radius 1. Circle D is tangent to both circle C and the x axis. If (x,y) is the center of D , $y < 2$, then y equals

- (a) $(x^2 + 1)/2$ (b) $(x^2 + 3)/6$ (c) $(x^2 + 2)/3$ (d) $(x^2 + 2)/6$ (e) $(x^2 + 1)/3$

30. If a man tosses a coin until he obtains either 3 heads or 3 tails, what is the probability he will need 5 tosses?

- (a) $1/4$ (b) $3/8$ (c) $5/8$ (d) $7/16$ (e) $11/32$