

David Essner Exam 28 2008-2009

1. The average score of a student on 5 exams was 70; when the lowest grade was dropped (not counted) the average score was 74. What was the lowest grade?  
(a) 50 (b) 54 (c) 56 (d) 57 (e) 59
2. If two sides of an isosceles right triangle each have length 2 then what is the length of the altitude to the hypotenuse?  
(a)  $\sqrt{2}$  (b)  $\sqrt{3}$  (c)  $3/2$  (d)  $3/\sqrt{2}$  (e)  $2/\sqrt{3}$
3. John invests \$10,000 at 6% compounded once per year and George invests \$10,000 at 6% compounded twice per year. At the end of one year, George's investment is worth how much more than John's?  
(a) \$9 (b) \$12 (c) \$15 (d) \$18 (e) \$36
4. Towns A,B,C,D all lie in a straight line. Towns A,B are between towns C and D and A is between B and C. Letting  $d$  denote the distance between towns, if  $d(A,B) = 4$ ,  $d(A,D) = 3d(A,C)$ ,  $2d(B,C) = d(B,D)$  then  $d(C,D)$  equals  
(a) 24 (b) 30 (c) 36 (d) 42 (e) 48
5. A baseball league consists of four teams: A,B,C,D. Each team plays each other team four times. If A wins  $1/6$  of its games, B wins 7 games and C wins  $1/2$  of its games, then how many games does D win?  
(a) 6 (b) 7 (c) 8 (d) 9 (e) 10
6. There are how many integers  $N$ ,  $10 \leq N \leq 99$ , such that  $N$  equals seven times the sum of its two digits?  
(a) none (b) 1 (c) 2 (d) 3 (e) 4
7. Initially Jar A has 8 pounds of water and Jar B has  $x$  pounds of grain. Then 4 pounds of water are transferred from A to B, thoroughly mixed with the grain, and 10 pounds is transferred from B to A. If the final mixture in A has 6 pounds of water then  $x$  equals  
(a) 10 (b)  $25/2$  (c)  $40/3$  (d) 16 (e) 18
8. A square is inscribed in a semicircle of radius 1. The area of the square is  
(a)  $4/5$  (b)  $3/4$  (c)  $3\pi/4$  (d)  $4\pi/5$  (e)  $1/2$
9. In a particular election, everyone votes for either candidate A or candidate B. After 60% of the votes are counted, candidate A is leading 60% to 40%. What percentage of the remaining votes must B get in order to have the same number of votes as A?  
(a) 60 (b) 65 (c)  $66\frac{2}{3}$  (d) 70 (e) 75
10. If  $2/x + 3/y = 21$  and  $4/x - 1/y = 7$  then  $y/x$  equals  
(a) 6 (b)  $7/3$  (c)  $-7/3$  (d)  $3/5$  (e)  $8/5$

11. The largest positive integer  $n$  such that  $n^{80} < 7^{120}$  is  
 (a) 12 (b) 18 (c) 21 (d) 24 (e) 30
12. The midpoints of the vertices of a triangle are (1,2), (3,4) and (5,0). The sum of the  $x$  coordinates of the three vertices is then  
 (a) 4 (b) 6 (c) 7 (d) 9 (e) 17/2
13. If  $m > 0$  and  $c > 0$  then for what value of  $m$  will the lines  $y = c$ ,  $y = mx$ ,  $y = -mx$  form an equilateral triangle?  
 (a) 2 (b)  $\sqrt{2}$  (c)  $\sqrt{3}$  (d) the value depends on  $c$   
 (e) there is more than one value
14. A circle passes through the vertices of a triangle with sides of length 3,4,5. The radius of the circle is then  
 (a) 4/3 (b) 5/3 (c) 3/2 (d) 5/4 (e) 5/2
15. If  $a, b, c$  are positive integers and  $16/5 = a + \frac{1}{b + \frac{1}{c}}$  then  $a + b + c$  equals  
 (a) 5 (b) 8 (c) 10 (d) 12 (e) 17
16. If the third number of a geometric progression is 4, then the product of the second and fourth number is  
 (a) 8 (b) 12 (c) 16 (d) 24 (e) not solvable from the given information
17. There are how many integers  $n$ ,  $n > 3$ , such that  $(n - 3)$  divides  $(n^2 - n)$ ?  
 (a) 1 (b) 2 (c) 3 (d) more than 3 but a finite number (e) infinitely many
18. What is the value of  $\log_5 8^x$  where  $x = \log_2 5$ ?  
 (a) 3 (b) 2 (c) 5 (d) 5/8 (e) 8/5
19. Which range of numbers, including the end values, has the greatest sum?  
 (a) 100 to 179 (b) 200 to 259 (c) 300 to 339 (d) 400 to 429 (e) 500 to 519?
20. If  $i$  is the complex number such that  $i^2 = -1$ , then  $i^{123}$  equals  
 (a) 1 (b)  $-1$  (c)  $i$  (d)  $-i$  (e)  $1 + i$
21. What is the distance between opposite sides of a regular hexagon whose sides have length 1?  
 (a)  $\sqrt{2}$  (b)  $\sqrt{3}$  (c)  $(\sqrt{3} + 1)/2$  (d)  $\sqrt{6}$  (e)  $(\sqrt{6} - 1)/2$
22. If  $ab > 0$  then the two equations  $x + y = a$  and  $1/x + 1/y = b$  have a unique simultaneous solution for  $x$  and  $y$  provided  $ab$  equals  
 (a) 1 (b) 2 (c) 4 (d) 8 (e) 9

23. A train goes from town A to town B. If it averages 50 mph (miles per hour) then it will be 20 minutes late, and if it averages 80 mph then it will be 10 minutes early. If it goes 60 mph it will be (in minutes)  
 (a) early by 3 (b) early by  $7/3$  (c) late by 5 (d) late by  $15/2$  (e) late by  $20/3$
24. What is the sum of all solutions of  $||x - 1| - |x - 8|| = 3$ ?  
 (a) 3 (b) 5 (c) 7 (d) 9 (e) 11
25. A ladder rests vertically against a wall. If the foot of the ladder is pulled away from the wall along a horizontal surface then the midpoint of the ladder assumes what kind of path?  
 (a) straight line (b) circular (c) parabolic (d) elliptical (e) hyperbolic
26. Three men and three women are assigned different numbers selected at random from the integers 1 through 6. What is the probability the three numbers assigned to the men are all greater than the three numbers assigned to the women?  
 (a)  $1/6$  (b)  $1/9$  (c)  $1/12$  (d)  $1/18$  (e)  $1/20$
27. If  $f(x,y)$  satisfies for all numbers  $x,y$  the two equalities  $f(x,0) = x$  and  $f(x, y + 1) = f(f(x,y),y)$  then  $f(2,1)$  equals  
 (a) 0 (b) 1 (c) 2 (d) 3 (e) 4
28. If  $r,s$  are the roots of the equation  $x^2 + x + 3 = 0$  then  $1/r, 1/s$  are the roots of the equation  
 (a)  $x^2 - 3x + 1 = 0$  (b)  $x^2 + 3x - 1 = 0$  (c)  $6x^2 - x + 3 = 0$   
 (d)  $3x^2 + x + 1 = 0$  (e)  $6x^2 - 3x - 1 = 0$
29. There are how many positive odd integer divisors of  $9!$  ( $= 1 \times 2 \times 3 \times \dots \times 9$ )?  
 (a) 20 (b) 24 (c) 28 (d) 32 (e) 35
30. Tom has \$2 and Bill has \$1. A die is rolled and if it comes up 1 or 2 then Tom wins and Bill gives him \$1; if it comes up 3,4,5 or 6 then Bill wins and Tom gives him \$1. This is repeated until Tom or Bill has no money and the other is the victor. What is the probability that Tom is the victor?  
 (a)  $1/2$  (b)  $2/5$  (c)  $3/5$  (d)  $4/7$  (e)  $3/7$

