David Essner Exam 29 2009-2010

1. A golfer scored 72 on a round of golf. What score must he average on the next three rounds in order that he has an average of 70 for all four rounds?
   (a) 69 1/3 (b) 69 2/3 (c) 70 (d) 68 1/3 (e) 68 2/3

2. A can of mixed nuts consists of 30% peanuts, 20% almonds, 15% cashews, 15% walnuts, and the rest pecans. If all of the almonds are removed then the percent of pecans in the remainder is
   (a) 25 (b) 24 1/2 (c) 24 (d) 22 (e) 20

3. A car made a round trip between town *A* and town *B*. The average rate for the round trip was 50 mph (miles per hour) and the average rate from town *A* to town *B* was 60 mph. The average rate in mph from town *B* to town *A* was nearest what integer value?
   (a) 38 (b) 40 (c) 43 (d) 41 (e) 44

4. The last (unit) digit of 777^7 is
   (a) 9 (b) 7 (c) 6 (d) 3 (e) 1

5. How many times does the graph of $f(x) = 1 + (x-2)(x-4)(x-6)(x-8)(x-10)$ cross the x axis?
   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

6. The value of $3^{1003} + 3^{1002}$ divided by $3^{1001} - 3^{1000}$ is
   (a) 18 (b) 27 (c) 65 (d) 82 (e) 127

7. There are how many positive integer divisors of 210?

   (a) 7 (b) 10 (c) 12 (d) 16 (e) 21

8. Given that one of the roots of the equation $x^4 - 4x^3 + 4x^2 - 9 = 0$ is $x = 1 + \sqrt{2}\,i$, where $i^2 = -1$, then what is the sum of all the real roots of the equation?
   (a) 2 (b) 3 (c) 5 (d) 7 (e) there are no real roots

9. A box has 30 green balls and R red balls ($0 < R < 30$). How many additional red balls must be placed in the box so that 3/5 of all the balls in the box will be red?
   (a) $25 + R$ (b) $45 - R$ (c) $50 - R$ (d) $18 + R$ (e) $2R - 30$

10. There are how many prime numbers between 70 and 100?

    (a) 6 (b) 8 (c) 10 (d) 11 (e) 12

11. If c is a real number and the two roots of $x^2 - 2cx + 2c^2 - 6c$ are real, what value of c gives the greatest difference between the roots?

    (a) -3 (b) -1 (c) 1 (d) 4 (e) 3

12. An amount of money is invested at a fixed rate and compounded annually. If the value of the investment triples in 15 years, then in how many years would the value double?

(a) 10 (b) 15 $\log_{10} 2/3$ (c) 15 $(\log_{10} 2)/\log_{10} 3)$ (d) $(2/3)^{15}$ (e) depends on the rate

13. A group of six soccer players decide to play a match by dividing themselves into two teams of three players each. How many different matches are possible?

(a) 6 (b) 10 (c) 12 (d) 18 (d) 20

14. If an isosceles triangle has 2 sides of length 2, what is the maximum possible area of the triangle?

(a) 1 (b) 2 (c) $\sqrt{3}$ (d) $2\sqrt{2}$ (e) $\sqrt{2}$

15. There are how many different isosceles triangles whose sides have integral length and whose perimeter is 100?

(a) 12 (b) 16 (c) 20 (d) 24 (e) 48

16. A and B play a series of games; the winner of each game has probability 4/5 of winning the next game. If the probability A wins the first game is 3/4 and the probability B wins the first game is 1/4 then the probability that A wins the second game is

(a) 11/20 (b) 13/20 (c) 17/20 (d) 53/80 (e) 57/80

17. Let O be the center of circle of radius 1, and P, Q be points on the circle such that $\theta = \angle POQ$ is an acute angle. If R is a point outside the circle such that $OPRQ$ is a parallelogram then what is the area of the part of the parallelogram that is outside the circle?

(a) $\pi \cos \theta$ (b) $\cos \theta - \theta$ (c) $\sin \theta - \theta/2$ (d) $\sin \theta - \pi\theta$ (e) $2 \cos \theta - \theta/2$

18. If the line y = 2 - x is tangent to the circle C at the point (1,1) and the point (3,2) is on C, then the x coordinate of the center of C is

(a) 2 (b) 11/6 (c) 3/2 (d) 9/4 (e) 7/3

19. Given an arithmetic sequence of positive integers, if the sum of the first ten terms equals the 58^{th} term then the least possible value of the first term is

(a) 3 (b) 4 (c) 6 (d) 7 (e) 11

20. A bag has two red balls and one green ball. Three times a ball is drawn at random from the bag and then placed back into the bag. What is the probability that a red ball was drawn at least two of the three times?

(a) 2/3 (b) 5/9 (c) 7/9 (d) 16/27 (e) 20/27

21. Each entry in a Science Fair project receives a gold, silver or white ribbon. There are three times as many white ribbons as gold ribbons, and forty per cent of the ribbons are either gold or silver. What fraction of the ribbons is silver?

(a) 3/20 (b) 1/4 (c) 1/5 (d) 3/10 (e) 2/15

22. The double inequality $3/13 < N/39 < 21/65$ is true for how many integers N?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

23. A basketball player makes x% of his first n shots and then makes 1 of his next 2 shots. He has then made what percent of his $n + 2$ shots?

(a) $x + (n + 1)/(n + 2)$ (b) $x + (n + 1)/100(n + 2)$ (c) $n(\mathrm{x} + 1)/(n + 2)$
(d) $[100nx + 1]/(n + 2)$ (e) $(nx + 100)/(n + 2)$

24. If N is a perfect square, and M is the smallest perfect square larger than N, then M equals

(a) $N + \sqrt{N}$ (b) $N + \sqrt{N} + 1$ (c) $N + 2\sqrt{N}$ (d) $(N + 1)^2$
(e) $N + 2\sqrt{N} + 1$

25. William has a sales job that each month pays a salary of \$$S$ and a commission of \$$C$ for each item that he sells in excess of N items. If he sells more than N items for the month and earns \$$D$ then N equals

(a) $(D + S + NC)/C$ (b) $(D + S – NC)/C$ (c) $(D - S + NC)/C$
(d) $(D + S – C)/NC$ (e) $(D - S + C)/NC$

26. If $|x - 1| < .001$ then for $x \neq 1$ the value $\mathrm{y} = (x^2 + 3x - 4)/(x^2 – x)$ must satisfy

(a) $y \leq 0$ (b) $0 < y \leq 2$ (c) $2 < y \leq 4$ (d) $4 < y \leq 6$ (e) $y > 6$

27. The sum of all positive numbers x such that $(\log_x 3)(\log_x 9) + 2 = 5 \log_x 3$ is a value between

(a) 3 and 5 (b) 5 and 7 (c) 7 and 9 (d) 9 and 11 (e) 11 and 13

28. If x,y,z are all positive real numbers and

$$xy – 2x - 3y + \mathrm{z} + 5 = 0;\quad 3x – z = 8;\quad 2y + z = 5$$

then x equals

(a) 3 (b) 2 (c) 1 (d) 4 (e) 6

29. Given the three numbers: $A = 999!$ (= 1x2x...x999); $B = 500^{999}$; $C = 999^{500}$ then

(a) $A > B > C$ (b) $B > A > C$ (c) $A > C > B$ (d) $B > C > A$ (e) $C > A > B$

30. Given concentric circles C1 and C2 with radii 4 and 6 respectively, a chord of C2 is divided into three equal length segments by its points of intersection with C1 and C2. The length of the chord is

(a) $3\sqrt{10}$ (b) 9/2 (c) $\sqrt{30}$ (c) 9 (e) $6\sqrt{5}$