David Essner Exam 30 2010-2011

1. Thirty students took a test. The average of all students was 70, the average of all students that passed was 75, and the average of all that failed was 60. How many students failed the test?

(a) 4 (b) 5 (c) 6 (d) 8 (e) 10

2. During a sale, product A was marked down 6% and product B was marked down 10%. If the marked down prices of A and B were the same, what was the ratio of the original prices of A and B?

(a) 3 to 5 (b) 5 to 3 (c) 45 to 47 (d) 50 to 53 (e) 47 to 53

3. If a,b are positive integers and the two roots of $x^2 - ax + b = 0$ are successive positive integers then a^2 equals

(a) $4b + 1$ (b) $b + 1/2$ (c) $b + 4$ (d) $(4 + b)/2$ *(e)* $(2b + 1)/4$

4. If the first three terms of a geometric series have the form x, $2x + 1$, $4x$ then

(a) $x < 0$ (b) $0 < x < 3/11$ (c) $3/11 < x < 2/3$ (d) $2/3 < x < 3$ (e) $x > 3$

5. If $1/\log 2 + 1/\log 8 = 1/\log c$ then (where the log base is a fixed number greater than 1) c equals:

(a) $3^{1/2}$ (b) 4/3 (c) $3^{3/2}$ (d) $2^{3/4}$ (e) 9/4

6. If $m > 0$ and $y = mx - 16$ intersects the parabola $y = x^2$ in exactly one point then m equals

(a) 1 (b) 2 (c) 4 (d) 6 (e) 8

7. If three rectangular faces of a box have areas 2,3,4 then the volume of the box is

(a) $3\sqrt{2}$ (b) $2\sqrt{3}$ (c) $2\sqrt{6}$ (d) $2\sqrt{2}$ (e) $\sqrt{5}$

8. Persons A and B play three games of chess. In each game the probability A wins is 1/2, the probability B wins is 1/4, and the probability of a draw (tie) is 1/4. What is the probability that A wins exactly 2 games?

(a) 3/8 (b) 1/2 (c) 11/32 (d) 5/16 (e) 7/16

9. An equilateral triangle ΔABE, with sides of length 1, is placed on top of a rectangle ABCD where AB, DC each have length 1 and BC, DA each have length 2. If x is the length of the line segment DE then what is the value of x^2?

(a) $5 + 2\sqrt{3}$ (b) $5 + 3\sqrt{2}$ (c) $5\sqrt{3}$ (d) $4 + 3\sqrt{2}$ (e) $2 + 4\sqrt{3}$

10. A rectangle has sides of length x and y. If the area A is unchanged when x is increased by 1 and y decreased by 4, and is also unchanged when x is decreased by 3 and y is increased by 2 then

(a) $A = 3/5$ (b) $A = 144/25$ (c) $A = 2$ (d) $A = 5/2$ (e) no such rectangle is possible

11. N is a positive integer that has remainder 41 when divided by 60. If N is divided by 12 the remainder is

(a) 3 (b) 4 (c) 5 (d) 7 (e) dependent on N

12. In a 1 mile race John finished 1/10 mile ahead of Bill, and Bill finished 1/20 mile ahead of Tom. If all three ran at a constant rate, John finished how many miles ahead of Tom?

(a) 3/20 (b) 9/100 (c) 29/200 (d) 3/25 (e) 1/10

13. If for all real numbers x, $f(2x + 1) = 6x + 8$ and $f(x) = Ax + B$ then B equals

(a) 1 (b) 2 (c) 4 (d) 5 (e) 7

14. All 150 students at a middle school take either math or history, and some take both. If twice as many students take math as history and 24 students take both, then how many students take math?

(a) 86 (b) 94 (c) 100 (d) 108 (e) 116

15. If $i^2 = -1$ then the quotient $(1 + i)^{11}/(1 - i)^{10}$ equals

(a) 1 (b) $2(1 + i)$ (c) $-(1 + i)$ (d) $2(1 - i)$ (e) i

16. If $a > 0$ then the values of a such that $|x^2 + 3x - 1| = a$ has two positive real valued solutions are described by

(a) $a > 9/4$ (b) $2 < a < 10$ (c) $3 < a < 10$ (d) $a < 9/2$ (e) $a < 1$

17. A quantity of acid is added to 10 gallons of a 20% solution of acid in water to give a 25% acid solution. How many gallons of water must be added to the resulting solution to return to a 20% acid solution?

(a) 7/2 (b) 8/3 (c) 13/4 (d) 11/3 (e) 11/4

18. Assuming investments are compounded annually at some rate r, if an amount \$100 earns \$100 interest in 10 years then what sum S in dollars earns \$200 interest in 20 years?

(a) $100\sqrt{2}$ (b) 100 (c) 75 (d) 200/3 (e) 80

19. A circle of radius 1 and a square have the same center and equal areas. If the circle intersects one side of the square at points P and Q, and x is the length of the segment PQ, then x^2 equals

(a) $(\pi - 1)/4$ (b) $(\pi - 2)$ (c) $(\pi - 1)/2$ (d) $(4 - \pi)$ (e) $(\pi + 1)/4$

20. For how many different positive integers n does $\sqrt{n}$ differ from $\sqrt{100}$ by less than 1?

(a) 1 (b) 9 (c) 11 (d) 38 (e) 39

21. Let ^ denote the symbol for exponent i.e. $a\text{^}b = a^b$. If $x_1 = 2$ and $x_{n+1} = 2\text{^}\, x_n$ for n = 1,2,3, … then the smallest value of n such that $x_n > 10^{10,000}$ is

(a) 5 (b) 7 (c) 11 (d) 20 (e) 40

22. In the Cartesian plane the point (1,2) is shifted 3 units to the right to the point P, the point P is rotated 90° counter clock-wise about the origin to the point Q, and Q is then reflected about the x axis to the point (a,b). The sum $a + b$ equals

(a) -8 (b) -6 (c) –9/2 (d) –7/2 (e) –4

23. A certain unfair coin has unequal probabilities for the two possible outcomes of obtaining a head or tail. If the coin is tossed twice what is the probability p that the second toss yields a different outcome from the first?

(a) $p > 1/2$ (b) $p < 1/2$ (c) $p = 1/2$
(d) p could be any number between 0 and 1
(e) none of (a)-(d) is necessarily true

24. There are how many integer pairs (x,y) such that $1 \le x < y \le 60$?

(a) 1600 (b) 1680 (c) 1720 (d) 1770 (e) 182 0

25. The sum of all real positive number solutions of $x^4 - 6 = 5x(x^2 - x - 1)$ is

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

26. If the distance is $\sqrt{2}$ between the centers of two circles having radius 1, then the area of the region bound by the intersection of the two circles is

(a) $\pi/2 - 1$ (b) $(\pi - \sqrt{2})/2$ (c) $(2\pi - 1)/4$ (d) $(2\pi - \sqrt{2})/4$ (e) $(\pi + 1)/4$

27. Given $\theta = 3\pi/8$ radians, $A = \sin\theta$, $B = \cos\theta$, and $C = \cot\theta$ then

(a) $B < C < A$ (b) $A < C < B$ (c) $B < A < C$ (d) $C < A < B$ (e) $C < B < A$

28. In the Cartesian plane what is the value of the area of the triangle determined by the inequalities $y \ge 2x$, $y \ge -x$ and $y \le x + 4$?

(a) 17/2 (b) 12 (c) 14 (d) 16 (e) 31/2

29. One leg of a right triangle has length 3. If the length of the altitude to the hypotenuse is 2 and x is the length of the hypotenuse then x^2 equals

(a) 13 (b) 39/2 (c) 15 (d) 81/5 (e) 122/7

30. In a population of 601,000 there are 1,000 who have a certain disease. A test for the disease is given to each person. The test gives a correct (positive) result for 96% of those

who have the disease and a correct (negative) result for 99% of those who do not have the disease. Of those who get a positive result, what fraction best approximates those who have the disease?

(a) 2/7 (b) 1/7 (c) 3/7 (d) 5/7 (e) 17/19