Preliminary Exam

1. Tom, Alice and John took an exam. Alice scored 80. Tom scored 10 more than the average of the three, while John scored 16 less than the average of the three. The average of the three was then

    (a) 72   (b) 74   (c) $75\frac{2}{3}$ (d) $76\frac{1}{3}$ (e) 78

2. Two sides of an isosceles triangle have length 2 and 5. What is the area of the triangle?

    (a) 5   (b) $2\sqrt{6}$ (c) $\sqrt{21}$ (d) $2\sqrt{5}$ (e) There is more than one possible value.

3. What is the sum $1-3+5-7+9-\ldots+81$? (The terms increase in magnitude by 2 and alternate in sign.)

    (a) -15 (b) 1   (c) 27   (d) 39   (e) 41

4. What is the area of a rectangle if the diagonals have length 1 and $60°$ is an angle of their intersection?

    (a) $1/2$ (b) $\sqrt{2}/2$ (c) $\sqrt{3}/2$ (d) $\sqrt{3}/4$ (e) $(\sqrt{3}+1)/2$

5. A multiple choice test has 30 questions and 5 choices for each question. If a student answers all 30 questions and the score is [number right - (number wrong/4)] then which of the following is a possible score?

    (a) -10 (b) 5.25   (c) 7.75   (d) 8.75   (e) 9.25

6. Line L_1 has slope $1/2$ and Line L_2 has slope $1/3$. If L_1 and L_2 have the same y–intercept b and the sum of the x–intercepts of L_1 and L_2 is 10, then b equals

    (a) $5/6$ (b) $-6/5$ (c) -2 (d) $3/2$ (e) 2

7. What is the value of $(\log_2 3)(\log_3 4)$?

    (a) $3/4$ (b) $4/3$ (c) $3/2$ (d) 2   (e) $8/3$

8. If $0 < x < \pi/2$ and $\sin x = 2\cos x$ then $(\sin x)(\cos x)$ equals

    (a) $1/3$ (b) $2/5$ (c) $1/5$ (d) $3/8$ (e) $\sqrt{3}/4$

9. How many positive integer pairs (m, n) satisfy the equation $2m + 7n = 835$?

    (a) 44   (b) 51   (c) 60   (d) 71   (e) 119

10. From a point P two tangent lines are drawn to a circle C. If A, B are the tangent points, O is the center of C, and $\angle APB = 30°$ then $\angle AOB$ equals

    (a) $60°$ (b) $90°$ (c) $120°$ (d) $150°$ (e) $180°$

11. Let the function f satisfy $f(xy) = f(x)/y$ for all positive numbers x, y. If $f(5) = 10$ then what is the value of $f(8)$?

    (a) 4   (b) $16/5$ (c) $32/5$ (d) $25/4$ (e) There are many possible answers.

12. A full glass contains a mixture of $4/5$ water and $1/5$ alcohol. First $1/4$ of the content is removed and replaced with alcohol. Then $1/3$ of the resulting mixture is removed and replaced with alcohol. What fraction of the final mixture is alcohol?

    (a) $13/20$ (b) $11/20$ (c) $2/5$ (d) $3/5$ (e) $47/60$

13. If a, b are real numbers, $a + b = 3$ and $a^2 + b^2 = 45$ then the value of $a^3 + b^3$ is

    (a) 27   (b) 87   (c) 189   (d) 135   (e) not uniquely determined.

14. Which of the following sets of three numbers form the lengths of the sides of an obtuse triangle?

    (a) $5, 6, 12$ (b) $4, 6, 7$ (c) $4, 5, 6$ (d) $3, 4, 5$ (e) $2, 3, 4$

15. In the Cartesian plane points P, Q are respectively in the first and fourth quadrants and each is a distance of 10 from the origin O. If segments OP, OQ respectively make an angle of $40°$, $20°$ with the positive x–axis and M is the midpoint of PQ, then the distance from O to M is

(a) $4\sqrt{2}$ (b) $5\sqrt{3}$ (c) 5   (d) $5\sqrt{2}$ (e) $4\sqrt{3}$

16. What is the real number value of x such that $2^x + 4^x = 12$?

(a) $\log_2 3$ (b) $\log_3 2$ (c) $\log_3 4$ (d) $\log_4 3$ (e) $\log_2 12$

17. The length of a chord of a circle is 14 and the shortest distance from the midpoint of the chord to the circle is 5. Then the radius of the circle equals

(a) $28/5$ (b) $37/5$ (c) $28/3$ (d) $19/2$ (e) $2\sqrt{6}$

18. In a race A, B, and C run at a constant speed. When A finishes, B is 20 feet behind and C is 29 feet behind. When B finishes, C is 10 feet behind. What was the distance of the race?

(a) 100 feet   (b) 150 feet   (c) 200 feet   (d) 250 feet   (e) 300 feet

19. Three standard six-sided dice are rolled. What is the probability that the largest number that occurs is a five?

(a) $16/216$ (b) $25/216$ (c) $48/216$ (d) $61/216$ (e) $75/216$

20. How many non-congruent rectangles are there which satisfy both (i) the lengths of the sides are integers and (ii) the perimeter equals the area?

(a) 0   (b) 1   (c) 2   (d) 3   (e) more than 3

21. If the polynomial $P(x) = x^3 - 4x^2 + ax + 30$, where a is a real number, has roots $2, r, s$, then what is $|r - s|$?

(a) 2   (b) 4   (c) 6   (d) 8   (e) There is more than one possible value.

22. Mr. Jones invested the amount $\$P$. During the first year the value of the investment increased by $x\%$ and during the second year it decreased by $y\%$ of the amount at the end of the first year. If the value of the investment was exactly $\$P$ at the end of the second year then x equals

(a) y (b) $101y/100$ (c) $100y/101$ (d) $100y/(100 - y)$ (e) $101y/(101 - y)$

23. How many numbers between 1 and 1000 are integer multiples of either 6 or 15?

(a) 265   (b) 232   (c) 221   (d) 199   (e) 33

24. Three men and three women are assigned different numbers selected at random from the integers 1 through 9. What is the probability the three numbers assigned to the men are all greater than the three numbers assigned to the women?

(a) $1/2$ (b) $1/6$ (c) $1/9$ (d) $1/20$ (e) $1/120$

25. If $y = x^2 - 2x$, what is the sum of all distinct real numbers x such that $(y + 2)^y = 1$?

(a) -1 (b) 1   (c) 2   (d) 3   (e) $3/2$

26. The difference $\sqrt{10} - 3$ is nearest to which of the following fractions?

(a) $1/3$ (b) $3/10$ (c) $2/9$ (d) $1/4$ (e) $1/6$

27. In the Cartesian plane if a line through the point $(10, 0)$ is tangent to the circle $x^2 + y^2 = 25$ at the point (a, b) then a equals

(a) $5/2$ (b) $5/\sqrt{2}$ (c) $5/\sqrt{3}$ (d) $5\sqrt{3}/2$ (e) $12/5$

28. If $50! = N(10^k)$, where N is not a multiple of 10, then k equals

(a) 5   (b) 7   (c) 12   (d) 15   (e) 20

29. At a high school $2/5$ of the students are boys and $1/3$ of the seniors are boys. If $1/5$ of the boys are seniors then what fraction of the girls are seniors?

(a) $7/25$ (b) $3/10$ (c) $6/25$ (d) $4/15$ (e) $1/4$

30. The ratio $\dfrac{(1 + i)^{10}}{(1 - i)^7}$, where $i^2 = -1$, equals

(a) $4 - i$ (b) $2 + 2i$ (c) $1 - 2i$ (d) $3 - 3i$ (e) $4i - 1$