1. (d) \( \log_a a^{10} = \log_a x^2 \) gives \( 10 = 2 \log_a x \)

2. (c) By similar triangles \( \frac{AC}{AE} = \frac{BC}{DE} = 3/4 \); hence \( AC = 7 \frac{1}{2} \)

3. (b) Four times each year the interest is \((12/4\%) = 3\%\) of the value, and each time the new value is the old value multiplied by \(1.03\).

4. (d) The sum = \( 10 + 12 + \ldots + 100 = \left( \frac{10 + 100}{2} \right) \times 46 \). Note \((10 + 100)/2\) is the average term and 46 is the number of terms

5. (b) \( n!/(n - 2)! = \left[ 1 \times 2 \times \ldots \times (n - 2) \times (n - 1) \times n \right] / \left[ 1 \times 2 \times \ldots \times (n - 2) \right] = (n - 1) \times n \).

6. (d) The total score for the first 3 exams is \(3 \times 90 = 270\) and for all 5 exams is \(5 \times 88 = 440\) leaving \( 170 \) for the last two exams.

7. (c) \( 210 = 1024 - 103 \) so \( 240 = (2^{10})^4 \approx (10^3)^4 = 10^{12} \)

8. (b) \( (x - 1)^2 + y^2 = x^2 + (y - 2)^2 \) by the distance formula.

9. (d) \( f(1) = 1 > 0 > -1 = f(2) \) so there is a root between 1 and 2.

10. (e) \( \sin \theta = 3/5 \), \( \cos \theta = 4/5 \) and \( \tan 2 \theta = \sin 2 \theta / \cos 2 \theta \).

11. (d) If \( d \) is the distance between \( A \) and \( B \), and \( x \) is the unknown speed then \( 2d/35 = d/30 + d/x \) is the total time. Solve for \( x \).

12. (d) \( e^{ix} = \cos \pi + i \sin \pi \)

13. (a) \( f(3) = f(2) - 2f(1) + f(0) = 4 - 6 + 2 = 0 \); \( f(4) = f(3) - 2f(2) + f(1) = 0 - 8 + 3 = -5 \); \( f(5) = f(4) - 2f(3) + f(2) = -5 + 4 = -1 \)

14. (b) All powers of \( x \) must be even or all must be odd.

15. (b) \( 3 \times 2 = 3x2 - 3 - 2 = 1 \); \((3 \times 2) \times 3 = (3 \times 2) \times 3 - (3 \times 2) - 3 \)

16. (e) \( x + 2 = 3(x - 2) \) or \( x + 2 = -3(x - 2) \)

17. (e) Amounts of alcohol \( A \) and water \( W \) at successive stages are \( 12W, 3A \rightarrow 12W \), \( 8A \rightarrow 9W \), \( 6A \rightarrow 9W \), \( 11A \rightarrow 27/4 \ W, 33A \).

18. (b) (See figure below) \( |AD| = |BD| = 1 \) and the area of the triangle \( ABC \)

19. (d) After \( n \) payments Ed has \( 1 + 2 + \ldots + 2^{n-1} = 2^n - 1 \) and Dona has \( 22 + 15n \).

   If \( n \geq 7 \) then \( 2^n - 1 \geq 22 + 15n \)

20. (d) Divide the numerator and denominator of the fraction by \( x^2 \). For \( x \) large \( 1 + 3/x + 12/x^2 \) is near 1 and \( 2 + 5/x + 3/x^2 \) is near 2, so the fraction is near 1/2.

21. (e) Let \( x = .32121212 \ldots \). Then \( 100 \times x - x = 32.121212 \ldots = 31.8 \) so \( x = 31.8/99 = 318/990 = 53/165 \). Another method is \( x = 3/10 + 21/1000 (1 + 1/100 + 1/(100)^2 + \ldots) \approx 3/10 + (21/1000)/(100/99) \)

22. (b) \( a_1 - b_1, a_2 - b_2, \ldots \) is also an arithmetic progression, so the sum is \( 3 + 4 + 5 + 6 + 7 + 8 + 9 = 42 \)
23. (See the figure) Triangles ABC and A'BC both satisfy the conditions of the problem, but have different areas.

24. (d) If the roots of the polynomial are \( r_1, r_2, r_3 \) then the polynomial is \((x - r_1)(x - r_2)(x - r_3)\), the constant term is \(-r_1r_2r_3\) and the coefficient of \(x^2\) is \((r_1 + r_2 + r_3)\).

25. (a) The expansion is the sum of terms \( C(5,n)(x^2)^n(-2/x^3)^5 \) where \(C(5,n) = 5!/(n!(5-n)!))\). For the constant term, \(2n = 3(5 - n)\) so \(n = 3\) and the coefficient is \(C(5,3)(-2)^3\).

26. (c) If \(s\) is the side of the square then the radius of \(C\) is \(s/2\) and the side of \(E\) is \((2)(s/2)(\sqrt{3}/2) = s\sqrt{3}/2\). The area of \(E\) is then \((\sqrt{3}/2)(s\sqrt{3}/2)(s\sqrt{3}/4) = 3\sqrt{3}s^2/16\).

27. (d) The inequality is equivalent to \(a^3b + ab^3 - a^4 - b^4 \geq 0\). Factoring gives \(-(a^3 - b^3)(a - b) \leq 0\). If \(a < b\) or \(a > b\) then \((a^3 - b^3)(a - b) > 0\).

28. (d) This may be easily seen by sketching the ellipse and parabola and noting the four points of intersection.

29. (e) \((1 + a)^b - 1 + ba\) if \(a\) is small, by the binomial theorem.

30. (c) The exponential \(2^x\) grows more rapidly than powers of \(x\) or logarithms.

31. (b) If the point is \((a,b)\) then \(b = 2a\) and the line from \((4,2)\) to \((a,b)\) should be perpendicular to \(y = 2x\), so 
\[
(2a - 2)/(a - 4) = -1/2.
\]

32. (d) See the figure. \(|HK| = 5 + (1/2 \times 3)\); \(|CK| = 1/2 \times 3 = |KG|\) and therefore \(|HG| = (169/4 + 9/4)^{1/2}\). 

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![Diagram](attachment:image.png)
33. (e) If \( x \) is the given number then
\[
\begin{align*}
x^2 &= (5 + 2\sqrt{6}) - 2[(5 + 2\sqrt{6})(5 - 2\sqrt{6})]^{1/2} + (5 - 2\sqrt{6})
\end{align*}
\]

34. (d) If \( a \) divides both \( n - 11 \) and \( n + 49 \) then \( a \) divides \( (n + 49) - (n - 11) = 60 \)

35. (c) \( a_3 = 7, \ a_4 = 15, \ a_5 = 31, \ldots \) In general \( a_n = 2^n - 1 \) so \( (a_n)^{1/\alpha} = (2^n - 1)^{1/\alpha} \approx (2^n)^{1/\alpha} = 2 \)
for \( n \) large.

36. (a) If \( p(x) \) is the polynomial then \( p(x) = p(-x) \) for all numbers \( x \), so if \( p(x) \) has a minimum at \( x = r \) then it has the same minimum at \( x = -r \).

37. (b) \( 12 = y^2 - 2xy = y(y - 2x) \) so \( y \) divides 1 and is 1, 2, 3, 4, 6 or 12. Of these only \( y = 6 \) makes \( x \) a positive integer.

38. (e) There are 13 denominations, so there are \( 13 \times \left(\frac{4 \times 3}{2}\right) = 78 \) possible pairs. The remaining 3 cards can be chosen in \( \frac{48 \times 44 \times 40}{6} \) ways so as to have no more pairs, giving \( 78 \times 8 \times 44 \times 40 \) hands.

39. (b) Only \( f_1 \) is additive

40. (b) \( Q_n = Q_0 r^n \), where \( Q_0 \) is the initial amount, \( Q_n \) the amount after \( n \) years, and \( r \) the growth rate. Since \( Q_8 = 4Q_0 = Q_0 r^8 \), \( r = 4^{1/8} = 2^{1/4} \). Solve \( 8 \sqrt{2} = 2^{n/4} \) for \( n \).