Answers and Brief Solutions to E1987

1. (c) Let F be the score on the first exam and d the difference on successive exams. Then $F + d/2 = 78$ and $F + d = 80$.
2. (a) The numerator is near 1 and the denominator is negative and near 0.
3. (d) Subtract equation 2 from twice equation 1 to get $-y - 3z = 7$ and add -2 times equation 1 to equation 3 to get $-y - 3z = t - 16$. The answer follows from $7 = t - 16$.
4. (d) This is the number of solutions of the equation $i + j + k = 6$ where i,j,k are non-negative integers. For $i = 0,1,2,3,4,5,6$ there are 7,6,5,4,3,2,1 solutions.
5. (e) $.3(x + y) = y$ and $.6(x + y + z) = y + z$; eliminate x and simplify.
6. (a) $y = f(x - 2)$ has the point (5,7) and $y = 3f(x - 2)$ has the point (5,21).
7. (a) Substitution of $y = x + B$ gives $x^2 + 2x + (x + B)^2 = 0$. This is a quadratic equation in x and setting the discriminant equal to 0 gives $B^2 - 2B - 1 = 0$. Solve for B.

8 (b) By the binomial theorem, $(1 + x)^{1/2} \approx 1 + x/2$ if x is small,

9 (d) There are $3^3 = 27$ possible outcomes and $3! = 6$ of these produce all three balls.

10. (d) 60 mod 31 ≡ 29, 29 mod 11 ≡ 7 and 46 mod 7 ≡ 4.

11.(b) Let d be the distance from A to B. Then the total distance driven is $3d$ and the total time is $d/10 + d/40 + d/50$. The average speed is the total distance divided by the total time.

12.(e) If n is the number of pens and c the cost then $n(1.05)c = (3 + n)c$.

13.(b) Summing the number of points whose x coordinates are 1,2,3,…,17 gives 9 + 8 + 8 + 7 + 7 +… + 1 + 1.

14.(c) If P is the amount of the investment then $3P = P(1 + r/2)^{20}$

15.(a) The numbers are in succession the following powers of 2: 32, 48, 24, 128, 48. Thus the answer is 2^{128-24}.

16 (d) Let r,s be the roots. Then $m = -(r + s)$ and $n = rs$. If n is odd then each of r and s is odd and m is even.

17. (e) If he wins the last two bets then he wins $2.

18. (d) By *II*, Q is true and P is false; only (d) is true in this case.

19. (a) Let d be the number of feet Bill runs; then Tom runs $d - 100$ feet and hence 10/9 $(d - 100) = d$.

20.(d) If x is the side opposite the 75° angle then, by the law of sines, $x/\sin 75° = 6/\sin 60°$. Apply $\sin 75° = \sin(45° + 30°) = \sin 45° \cos 30° + \cos 30° \sin 45°$.

21. (c) *I* is true since $S = a - (b - c) - d$ and *III* is true since $S = (a - b) + (c - d)$.

22. (a) $\log_8 3 = 1/A$ gives $\log_2 3 = 3/A$ and $\log_{16} 5 = B$ gives $\log_2 5 = 4B$; add $3/A + 4B$.

23. (b) $F(1) = F(2) = 1$; $F(3) = 0$; $F(4) = F(5) = -1$; $F(6) = 0$; $F(7) = F(8) = 1$ implies $F(n) = F(n - 6)$ for $n > 6$. Thus $F(1{,}000) = F(4)$.

24. (e) **Method I**: By a counting argument there are successively 1,8,21,20,5 words with 0,1,2,3,4 a's **Method II:** Let x_n be the number of n letter words using only a,b without two consecutive a's; then $x_n = x_{n-1} + x_{n-2}$ and $x_1 = 2$; $x_2 = 3$.

25.(c) The region is a rectangle bounded by the lines $y = x + 3$, $y = x - 3$, and $y = 2 - x$, $y = -2 - x$. The vertices are (5/2,-1/2), (1/2,-5/2),(-5/2,1/2),and (-1/2,5/2). The distances between opposite sides are $3\sqrt{2}$ and $2\sqrt{2}$.

26. (e) $x^2 + x + (1 - A) = 0$; using the quadratic formula set the discriminant $1 - 4(1 - A) = 0$.

27. (c) $1987 = 87 \times 22 + 19 \times 3 + 16$. Each reduction of the multiplier of 87 by 1 causes an increase in the multiplier of 19 by 5 and a decrease in the remainder by 8 since $87 = 19 \times 5 - 8$. Thus $1987 = 87 \times 20 + 19 \times 13$.
28. (c) $9! \approx 3.6 \times 10^5$; multiply this by $10^{11} \times 1.1 \times 1.2 \times \ldots 1.9 \times 2 \approx 6.7 \times 10^{12}$
29. (b) Starting with 1 as the smallest integer and increasing successively by 1 there are 10,9,9,8,8,7,7,…,1 possibilities.
30. (d) $s = rx$ and $c = 2r \sin(x/2)$; thus $s/c = x/(2 \sin(x/2))$.
31. (e) $x/1 = 4/y = z/5$ gives $xy = 4$ and $yz = 20$. Values $x = 4$, $y = 1$, $z = 20$ give the maximum.
32. (c) A congruent triangle may be placed in the xy plane with vertices (0,0), (0,2), (1,0). If (x,y) is the point common to the rectangle and hypotenuse then $y = -2x + 2$. Thus the area of the rectangle is $A = x(-2x + 2) = -2(x - 1/2)^2 + 1/2$.