ANSWERS AND BRIEF SOLUTIONS TO E1988

1. (c) If x, $x + 4$ are the other two numbers then $2x + 4 + 60 = 4(35)$ gives $x = 38$, x + 4 = 42.
2. (c) If $b \neq 0$ and $d \neq 0$ then the slopes of the lines are $-a/b$ and $-d/e$; setting $-a/b = -1/(-d/e)$ gives $ad + be = 0$. If $b = 0$ then also $d = 0$ and again $ad + be = 0$.
3. (c) Adding the first two equations gives $2x + 2z = 10$, or $x + z = 5$. Thus $r = 1$ by comparison with the third equation.
4. (e) The ratio depends on the relative values of $1/x$ and y. e.g. if $1/x = 10^{10}$ and $y = 10^{20}$ then the ratio is near –1/4.
5. (c) $1/x$ is near 10^{-3} and $1/y$ is near 10^{-8}. Thus $1/x - 1/y$ is near 10^{-3} and the result is near 10^3.
6. (a) The equation of the translated graph is $(x - 4)^2 + (y + 2)^2 = 10$. The values $x = 5$, $y = 1$ satisfies the equation.
7. (a) If a coin is tossed n times there are 2^n different possible outcomes. The least value of n such that $2^n > 52$ is $n = 6$.
8. (e) 'If P then Q' is false only if P is true and Q is false. In (e), 'not Q' is true and 'not(P or Q)' is false.
9. (d) The interest is \$90 and the withdrawl for tax is \$27. Thus the value is \$(1,000 – 90 + 27).
10. (d) Let t = Tom's time and b = Bill's time. Then $6b + 8t = 4$ and $t = 2b$. Thus $6b + 16b = 4$ gives $b = 2/11$.
11. (a) Since $CB = 1$, then (B,C) is one of the pairs (9,9), (1,1), (3,7), (7,3). Only the latter gives the correct product.
12. (e) Let a be the first term and r the ratio. Then $ar = 1/3$ and $ar^3 = 4/27$ give $r = 2/3$ and $a = 1/2$. The third term is $ar^2 = 2/9$.
13. (a) The regions are in 1-1 correspondence with sequences $c_1c_2c_3c_4c_5$ where each c_i is either 0 or 1 depending on whether or not points in the region are inside the circle C_i or not; there are 2^5 such sequences.
14. (b) From $x^n \log 2 = \log y$, $n \log x + \log\log 2 = \log\log y$, and $\log\log 2 = 0$; the result follows
15. (d) By the Binomial Theorem $(y - x)^{1/2} = y^{1/2} + 1/2\,(y)^{-1/2}(-x) + \ldots$
16. (a) The line with slope 3 through (a,b) has equation $y = 3x + c$ where $c = -3a + b$. Solving this simultaneously with $x^2 + y^2 = 1$ gives $10x^2 + 6cx + (c - 1) = 0$ which has a solution if $c \leq 10$.
17. (a) Each such integer must be the number 2x3x5x7 = 210 multiplied by either 1 or a prime number between 11 and 47 inclusive. There are 12 such numbers 1,11,13,17,19,23,29,31,37,41,43,47.
18. (b) S is the sum of the first four terms of a geometric progression whose sum is $\frac{1}{1-x}$; thus II is true. If $x = 1/2$ then I is false; if x is near 1 then S is near 4 and $\frac{7}{1+x}$ is near 7/2 making III false.
19. (e) $x = \frac{529 - 3y}{4}$. Among the integers 529 – 3y which are positive, those which are divisible by 4 are of the form $520 - 12n$ where $n = 1,2,3,\ldots,43$.

20. (b) –2,3,-2 is such a sequence of 3 integers. Any negative integer must be followed by a positive integer, and any positive integer can be followed by at most one integer.
21. (e) Person 9 walks ½ mile, person 8 walks 3(1/2) miles, person 7 walks $3(1/2)^2$ miles, etc. continuing to person 1 gives $(3)^4(1/2)^5$ miles.
22. (d) This is the number of combinations of 2 objects from 8, which is 28.
23. (d) By DeMoivre's Theorem the values for b are sin(90°/3) = 1/2, sin(450°/3) = -1/2 and sin(810°/3) = 0.
24. (e) The possible ordered selections are *RRR*, *RBR*, *BRR*, and BBR where *R* is the drawing of a red ball and B of a black ball. These have respective probabilities $(1/2)^3$, $(1/2)^2(3/5)$, $(1/2)(3/5)^2$, and (1/2)(2/5)(3/4) and their sum is 121/200.
25. (d) If the roots are r, s, and 3 then $r + s = 1$, $rs = -2$, and $c = rs + 3r + 3s$. The roots are -1, 2 and 3 giving c = 1.
26. (c) The solution is the number of sequences of 1,2,3 which add to 6.

    Method *I*: By enumeration there are 13 which start with 1, 7 which start with 2 and 4 which start with 3.

    Method *II*: If s is the number of sequences of 1,2,3 which add to n, then $s_1 = 1$, $s_2 = 2$, $s_3 = 4$ and $s_n = s_{n-1} + s_{n-2} + s_{n-3}$. In this difference equation s_{n-k} is the number of sequences that start with k and add to n, $k = 1,2,3$.
27. (b) Substituting $n – 1$ for n gives $f(n – 1) = f(n – 2) – 2f(n – 3)$. Substitution from this for $f(n – 1)$ in the original equation for $f(n)$ gives the result.
28. (d) I, *II* and *III* are true. For instance if $(M – N) = KD$ and $(P – Q) = LD$ then $MP = (QK + NL + KLD)D + NQ$ shows *II* is true. Also $D = 5$, $M = 6$, $N = 1$, $P = 7$, $Q = 2$ shows *IV* is not necessarily true.
29. (b) Using the law of sines, x sin 30°/sin 75° = sin 45°/sin 105°. Thus x =

    sin 45°/sin 30°.