ANSWERS AND BRIEF SOLUTIONS TO E1989

1. (d) The total score for the honor students is 80x30 = 2400 and for the math students is 86x16 = 1376. The total score for all is (30 + 16 – 10)x81 = 1376. The total duplicate score is 2400 + 1376 – 2916 = 860 for 10 students.
2. (a) If $x > 0$ the equation $x^2 – 7x + 3x = 0$ gives $x = 0,4$. If $x \le 0$ the equation $x^2 + 7x + 3x = 0$ gives $x = 0, -10$; the answer is 4 + (-10).
3. (c) $S = 7(2 + 3 + 4 + \ldots + 28) = (7)(28)(29)/2 – 7$.
4. (b) 9! = 362,880 and 10! = 3,628,800.
5. (c) $b*a = b + 2a$ and $a*b = a + 2b$; $b\#a = b + a – 2 = a\#b$; $(a*b)*c = a + 2b + 2c$ and $a*(b*c) = a + 2b + 4c$; $(a\#b)\#c = a + b + c – 4 = a\#(b\#c)$
6. (d) If a is the first term and r the ratio than $ar^2 = 3$ and $ar^6 = 48$. Thus $r = 2$ and a = 3/4.
7. (d) Of any three successive integers one must be divisible by 2 and one (perhaps the same) by 3; hence the product is divisible by 6.
8. (c) Multiply the numerator and denominator by x; if x is small then the new numerator is near 5 and the new denominator is near –2.
9. (e) (5/6)(2/3) = 5/6
10. (c) The logic principle is 'if p then q' is equivalent to '(not p) or q'.
11. (a) The third equation is the second equation minus the first equation so the equations are dependent. If y is eliminated from any two equations the result yields $x = 3z + 4$.
12. (b) The final mixture has $[(20)(.2) + (10)(.3)](x/30) + (10)(.4) = 7x/30 + 4$ ounces of alcohol and $x + 10$ total ounces. Solve $7x/30 + 4 = (.3)(x + 10)$.
13. (a) The altitudes meet 2/3 of the distance from the vertex; thus the altitude length = 3/2 and the area is $(2)(1/2)(3/2)(\sqrt{3}/2)$
14. (b) The exponents of x add to 8 with the combinations 3,3,2,0; 3,3,1,1; 3,2,2,1; 2,2,2,2 respectively with 12,6,12,1 possibilities.
15. (d) $(x + i)(x – i) = x^2 + 1$ divides the left side of the equation, and the quotient is $(2x - 3)(x + 4)$; thus 3/2 and –4 are the real roots.
16. (d) After cancellation of the common terms the sum is 1 + 1/2 - 1/10 – 1/11.
17. (b) By the binomial expansion, if x is small then $(8 + x)^{1/3} \approx 8^{1/3} + (1/3)(8^{-2/3})(x)$. Substitute $x = .0036$ to obtain the answer.
18. (e) Solve the equation $x^2 + (3x + 4)^2 = r^2$ for x using the quadratic formula. The discriminant is $D = 40r^2 – 64$; set $D = 0$ and solve for r.
19. (d) Solve $\log x + 2 \log y = 4$, $\log y – \log z = 3$ and $\log x + 3 \log z = -4$ to get x = 4, y = 2 and $z = 1/4$.
20. (e) Adding the probabilities of the favorable cases black, red and red, red gives (2/5((3/5) + (3/5)(2/4).
21. (b) $\cos 2x = 3 \cos^2 x – 1 = 1/4$ gives $\cos^2 x = 5/8$; $\cos 2x = 1 – 2 \sin^2 x = 1/4$ gives $\sin^2 x = 3/8$; $\tan^2 x = \sin^2 x/\cos^2 x$ gives the answer.
22. (e) The values of $f(n)$ are respectively 1, 2, 1/2, 1, 0, 0, undefined since $f(5) = 0$ and log 0 is undefined. Therefore $f(n)$ is undefined if $n > 5$.
23. (e) Bill can only break even if he wins the last two bets; John wins \$9 if *B*ill wins the first two bets.
24. (c) $(1 + x)(1 – y) – 1 = x – y – xy$. In (c), (d) the largest of the x, y terms is less than in the other cases; choose (c) since then xy and $x – y$ have the same sign.

25. (a) $f(50) = 2/7 + 1 = 9/7$.
26. (d) The longest side of the triangle is 21 or x. If $21^2 < 9^2 + x^2$ and $x^2 < 21^2 + 9^2$ then all angles are acute; this is true for $x = 19, 20, 21$ and 22.
27. (a) If 3 divides $n + 2$ then 3 also divides $(n + 2) + 3k$ for all integers k; thus 3 divides $n - 1$ which is a factor of $n^2 + 6n - 7$.
28. (c) If r is the annual rate of interest then $A^{10} = 2$ and $10 \log A = \log 2$ where $A = (1 + r/365)^{365}$; solving $A^x = 3$ gives the answer.
29. (a) After the first four terms in the sum, each term is divisible by 5 and the sum of the first four terms is 35 which is also divisible by 5.
30. (a) Letting x = $\angle BCD$ then also $x = \angle ABE$. From triangle BCD it is seen that $\tan x = 3/2$ and hence $\cos x = 2/\sqrt{13}$; also $\cos x = BE/4$ from triangle ABE. Therefore $BE/4 = 2/\sqrt{13}$