ANSWERS AND BRIEF SOLUTIONS TO E1991

1. (c) The sum of the scores is 3x65 + 2x80 + 4x90 = 715. Divide this value by 9 to get 80.2.
2. (a) Add the probabilities that the sum of both colored dice equals each of 2,3,4,…,12. This is $(1^2 + 2^2 + \ldots + 6^2 + \ldots + 2^2 + 1^2)/36$.
3. (a) The equations are dependent and may be reduced to an equivalent system of two equations. Substitute $x = a, y = b, z = c$ in these two equations and solve simultaneously with $a + b = 1/7$.
4. (b) Since $x = 1$ satisfies the equation, $x - 1$ is a divisor of the given polynomial, and this division equals $(x - 1)(x^2 + 1)^2$. Thus $x = 1$ is the only real root; other roots are i and $-i$.
5. (c) The left side of the equation simplifies to x^4; thus $x^4 = 3x^2$ from which $x = \sqrt{3}$. Note the left side is not defined for $x = 0$.

6. (e) There are 3 with the digits 0,0,5; 6 with 0,1,4; 6 with 0,3,2; 3 with 0,1,3 and 3 with 1,2,2

7. (b) For any logical proposition p, 'p and not p' is always false so the implication in (b) is always true.
8. (c) This answer follows since (2x5x3) mod 7 = 2.
9. (d) By the binomial approximation $\sqrt{4+a}$ is approximately $4^{1/2} + (1/2)(4)^{-1/2} a + (1/2)(-1/2)(4)^{-3/2}a^2$.
10. (c) If A is the amount of the investment then solve $3A = A(1 + r)^{10}$ for r.
11. (d) The original selling price is $1.5x$ and the sales price is $1.5x - (.1)(1.5x)$.
12. (a) Let a = the rate of Bill, b = the time of Bill, s = the rate of Tom and t = the time of Tom. Then $5 = ab = st$, $s = 4a/5$ and $b + t = 2$. Solve for a.
13. (c) The sequence is 0,1,1/2,3/4,5/8,11/16,21/32.43/64, … . . The numerator of $f(n)$ is $\frac{2^{n-1}}{3}$ and the denominator is 2^{n-1}
14. (e) The sequence is $2^1, 2^2, 2^4, 2^8, \ldots 2^{512}$. Since $2^{10} > 10^3$ it follows that $2^{512} > 10^{(51x3)}$.
15. (d) The amount of solution in jar *1* after the two pourings is $(10 - x)(.2) + \frac{x(5+.2x)}{10+x}$; set this equal to 3 and solve for x.

16, (b) If P is the point of intersection of AC and the altitude from B to AC then

$$AC = AP + PC = 2\sqrt{3} + 2$$

17. (d) If a is the first term and n the difference between each term and the next then $10a + (1 + 2 + \ldots + 9)n = 10 + 45n = 205$ and $a + 3n = 16$. Solve simultaneously for a.
18. (b) If Tom wins the first bet and Bill wins the next two bets then Bill wins $2.
19. (a) The quotient is near $(8/9)^n$ which is near 0 if n is large.
20. (e) Let (b,b^2) be the intersection point on the parabola. Then the equations $y = x^2$ and $y + a = \frac{(b^2+a)x}{b}$ have a single solution. Eliminate y and solve the resulting quadratic equation for x and get the discriminant $(b^2 - a)^2$; set this to 0 and get $b = \sqrt{a}$. The slope is $\frac{(b^2+a)}{b} = 2\sqrt{a}$.

21. (d) The number 3 divides the least common multiple of x and y if and only if 3 divides at least one of x and y. Thus count all pairs of which at least one is a 3, 5 or 9.
22. (e) (*I*) is true since $x^2 < y^4 < y^3$ and (*III*) is true since $x < y^2 < y$ and $y < z^{3/2} < z$. (*II*) and (*IV*) are false for the values $x = .6$, $y = .8$ and $z = .9$.
23. (b) Let $y = z + n$ and $x = z + 2n$; then substitute in the given equation and solve for x. This gives $x = 3n$, $y = 4n$ and $z = 5n$ for any positive integer n in the solution set.
24. (a) The graphs are the circles with center (0,2) and radius 2 and center (4,5) and radius $\sqrt{k+41}$. Since the distance between the centers is 5 it follows that $k + 41 = 9$.
25. (e) The altitude from *B* to *AC* would be of length 5, and this is more than the length of *AB*. Thus there can be no triangle with the given measurements.
26. (e) The quadrilaterals *ABEF* and *ABCD* are similar. Thus *BC/BE* = *AD/AF*, or $\frac{12}{x} = \frac{1+x}{1}$; solving gives $x = 3$.
27. (c) The shift gives the equation $(x - 3)y = 1$ and the rotation replaces x by $-y$ and y by x.
28. (b) The probability each team wins in exactly 5 games is$[4 \mathrm{x} \frac{1}{2} \mathrm{x} (\frac{1}{2})^3](\frac{1}{2}) = 1/8$ where 4 denotes the number of possible games for the one loss.
29. (a) There are 6 ways to pick the 5 winning numbers and for each of these ways there are 43 ways to pick the non-winning numbers.
30. (d) Multiply y by the conjugate $x + \sqrt{x^2 - 1}$ divided by itself to get $\frac{1}{x + \sqrt{x^2 - 1}}$ which is always decreasing and positive, and approaches 0 as x gets larger and larger.

Subscripts

$_1$ $_2$ $_3$ $_5$ $_N$ $_n$

Exponents

^{2}

Radicals

$\sqrt{2}$ $\sqrt{3}$ $\sqrt{5}$ $\sqrt{6}$ $\sqrt{10}$ $\sqrt{7}$ $\sqrt{30}$ $\sqrt{65}$ $\sqrt{ab}$ $\sqrt{x}$

$\sqrt{a}$ $\sqrt{17}$ $\sqrt{x^2 - 1}$

Math Symbols

≠ ≡ ≈ ∠ | ∪ ∩ ° ~ ≥ ≤

Greek Symbols

π α β δ ε ϕ π θ π

≅ ÷ ⊃ × □ Σ

Fractions

$$\frac{1}{2}$$