Answers and Brief Solutions to E1993

1. (e) Each of 10,20,2x5,12x15,22x25 is a multiple of 10.

2. (e) The probability they are equal is 1/6; the answer is 1/2(1 - 1/6).

3. (b) Solving $(a+b)/2 = 16$, $(a+c)/2 = 12$, $(b+2c)/2 = 19$ gives $a = 14$, $b = 18$, $c = 10$.

4. (c) If S is the selling price and C the cost then $S - C = 10$ and $.9S - C = 8$. Then $.9S$ is the new price.

5. (a) The contrapositive of the second statement is "If $x \geq 3$ then $y < 7$" which together with $x \geq 3$ implies $y < 7$; also $x \geq 3$ is true.

6. (b) Add -3 times equation 1 to equation 2 and -5 times equation 1 to equation 3. This gives $7y - 5z = -2a$ and $7y - 5z = -5a + b$. Solve $-2a = -5a + b$.

7. (d) Solve $\frac{5(.2)+.1x}{5+x} = .12$ for x.

8. (e) If r is Bill's rate then the time to run the race for Bill is $1000/r$ and for John is $600/.8r + 400/xr$. Set these equal and solve for x.

9. (c) Let Jack win x games and lose y games. Then $2y - 5 - x = 0$ or $2y - 5 - x = 1$. Thus $2y - x = 5$ or $2y - x = 6$ for positive integers x,y. If $x + y = n$ then $n = 3y - 5$ or $n = 3y - 6$ and n cannot be 5,8,11,14,etc.

10. (a) Both x and $\log_{10}x$ are positive but x is much larger.

11. (c) If $2n$, $2n + 2$, $2n + 4$ are consecutive even integers the sum is $6(n + 1)$. Thus we count perfect squares which are divisible by 6. Of these only 6^2, 12^2, 18^2 are less than 500.

12. (d) Solve $(1 + r/2)^2 A = 2A$ for r, where A is the amount of the investment.

13. (a) If the Yankees or Dodgers win the last game then there are $C(6,3) = 20$ ways, where $C(6,3)$ is the number of combinations of three objects from 6.

14. (d) From $10^c = b$ and $b^d = c$ obtain $10^{cd} = c$ from which the answer follows.

15. (b) By long division $4x^3 + 8x^2 - 11x - 15 = (2x - 3)(2x^2 + 7x + 5)$.
Thus the roots are 3/2, -5/2, and -1.

16. (a) By the Binomial Theorem $(1 + .01)^{10} \approx 1 + 10(.01) + 45(.01)^2 + 120(.01)^3$.

17. (c) If n is an even number then $a_n = \frac{1+1/3^{n/2}}{2}$. Thus if n is a large even number then a_n is near *1/2* and a_{n+1} is near 3/2.

18. (e) If C is the intersection point of side AB and the altitude then AC has length x and CB has length $\sqrt{3}\,x$. Thus $1 = x\frac{x+\sqrt{3}x}{2}$.

19. (a) By the Pythagorean Theorem $ED = \sqrt{3}$ and by similar triangles $BC = 3\sqrt{3}$
The area is $[3 \text{ x} 3\sqrt{3}]/2$.

20. (c) Solve simultaneously $y = x - 3$ and also $y = x + 3$ with $x^2 + y^2 = 29$ to get x ,y values -2,5 and 2,-5.

21. (b) For integers K,L we have $M + 2N = 6K + 4$ and $2M + N = 3L + x$. Add equations to get $3M + 3N = 3(2K + L + 1) + (1 + x)$. Thus 3 divides $(1 + x)$ and a solution is then $M = 2$, $N = 7$.

22. (d) The population at time t years after 1992 is $P = x2^{t/12}$ Set $t = 4$ to get the result.

23. (a) The ratio of the circumference to the diameter is π.

24.(c) Substitute to get $y + (y - 1)^2 = C$ or $y^2 - y + (1 - C) = 0$. Solve by the quadratic formula; the discriminant is $-3 + 4C$ and there is just one solution if this is 0.

25.(b) Answer (a) < 100 and answer (c) < 10^{18} . Answer (b) > answer (d) since
$4^{100} = 4^8 \text{ x } 4^{20} = 16^{40} \text{ x } 4^{20} > 10^{20} \text{ x } 3^{20}$ and > answer (e) since
$4^{100} > 4^{30} \text{ x } 4^{30} \text{ x } 4^{30} = 64^{10} \text{ x} 64^{10} \text{ x} 64^{10} > 10! \frac{20!}{10!} \frac{30!}{20!}$

26. (b) $S_n = 2^{n+1} - 1$. Note $2^{10} = 1024$ and $2^6 = 64$.

27. (a) Since $12 < x^2 yz < 20$ then $12/yz < x^2 < 20/yz$ and $2 < x^2 < 4$.
The values x = 1.5, $y = 2.1$, $z = 2.8$ show other cases are not necessarily true.

28. (b) The largest prime divisor is 17.

29. (e) The units digit of 7^n for $n = 1,2,3,4,5$ respectively is 7,9,3,1,7. Thus it is periodic of period 4 and since 50/4 has remainder 2 the remainder for $n = 50$ is the same as for $n =$

2.

30. (e) In the Figure $x = CF$, $y = FG$; 1/6 = area $BEC = (2x)/2$; $x = 1/6$; 1 = area $ABCD = 2(x + y)$; $y = 1/3$; Let $a = EC$. By similar triangles $\frac{a}{x} = \frac{a+1}{x+y}$ gives a = 1/2.