

## Answers and Brief Solutions to E1994

1. (c) The average is  $x = (cx + 2cx + 4cx + 8cx)/4$ . Find  $c$ .
2. (c)  $\sin 3\pi t = 0$  if  $t$  is an integer multiple of  $1/3$  minute (= 20 seconds). The values 6,4 do not affect the periodicity of the point.
3. (d)  $c = 3$  is not possible since then  $b + c < a$ ; (b) and (c) give many possible triangles.
4. (a) Both  $A$  and  $B$  are equivalent to  $a \geq 0, b \geq 0$  or  $a \leq 0, b \leq 0$ .
5. (d) The equation may be written  $y + 5a^2/4 = (x + a/2)^2$ .
6. (e) Since it is a right triangle the hypotenuse is a diameter.
7. (d)  $A$  can win two games before  $B$  in 3 ways with probabilities : (1)  $AA : (2/3)(2/3)$  (2)  $ABA : (2/3)(1/3)(2/3)$  (3)  $BAA : (1/3)(2/3)(2/3)$ . Add the three.
8. (b) By the Binomial Theorem  $(100^2 - 1)^{1/2} \approx 100 + (1/2)(1/100)(-1)$ .
9. (a)  $V = 100(1 + r/2)^4 + 100(1 + r/2)^2$ . Expand the binomials and drop terms which are powers of  $r$  greater than 1 since  $r$  is a small number.
10. (d)  $w = 3x + 2 = 21y + 17 = 168z + 80$  by substitution. Set  $z = 1$ .
11. (e)  $a^3 = 16 = 4^2 = b^{14}$ . Thus  $a = b^{14/3}$  and the answer is  $3/14$ .
12. (a) Jar  $C$  still has 20% solution after addition from jar  $B$ . Thus  $.10 = [.2x + (.05)(40)]/(x + 40)$ . Solve for  $x$ .
13. (e) If  $R$  is the rate then  $200 = R(100/r + 100/s)$ .
14. (d)  $\text{Sales} = 80x + 40y + 20(100 - x - y) > 5000 = \text{Cost}$ .
15. (b) By inspection 1 is a root. By division the left side is  $(x - 1)^2(x + \pi x + \pi)$ . The roots of the quadratic are not real.
16. (b) The first two equations give  $x = a - 1$  and the first and third give  $x = 1 - b$ . Equate these two.
17. (c) The line through (2,5) perpendicular to  $y = 2x - 3$  is  $y - 5 = (-1/2)(x - 2)$ . Solve for  $x, y$ .

18. (b) If  $p$  is the probability John wins then the probability Bill wins is  $1 - p$ . If Bill wins the first bet then his probability of winning becomes  $p$ . Thus  $1 - p = (1/2)p$ .

19. (a) If  $x < 1/3$  then  $y = -x - 4$ ; if  $1/3 < x < 1/2$  then  $y = 5x - 6$  and if  $x > 1/2$  then  $y = x + 4$ . Thus  $y$  decreases to the left of  $x = 1/3$  and increases to the right of  $x = 1/3$ ; the minimum is at  $x = 1/3$ .

20. (e) Let  $r, s$  be the respective ratios of the  $a, b$  sequences. Then  $a_1 b_1 s = 1$ ,  $a_1 b_1 r = 4/3$ ,  $a_1 b_1 s^2 = 1/2$ . Therefore  $s = 1/2$ ,  $r = 2/3$  and  $a_1 + 2/a_1 = 3$ . It follows that  $a_1 = 1$  and  $a_3 = 1(2/3)^2$ .

21. (c) There are 120 ways to pick three different numbers, 90 ways to select exactly two numbers the same, and 10 ways to select all three numbers the same.

22. (d). This is equivalent to  $(3/2)^n > 2^{10} = 1024$ . Since  $(3/2)^3 < 2^2$ , it follows that  $(3/2)^{15} < 2^{10}$ . Since  $(3/2)^2 > 2$  and  $(3/2)^4 > 5$  it follows that  $(3/2)^{18} = (3/2)^2 (3/2)^{16} > 2(5)^4 > 1024$ .

23. (b) If  $P(x)$  is expanded the coefficient of  $x^5$  is 32. If  $x$  is very large then  $P(x)/Q(x)$  is approximately the ratio of the coefficients of their highest powers when these powers are the same.

24. (a) The successive values are  $1, 3, 7, 15, \dots, 2^{n+1} - 1$ .

25. (e)  $P(x) = (x - 2)(x + 1)d(x) + (3x - 11) = (x - 3)(x + 1)e(x) + (2x + t)$  where  $d(x)$ ,  $e(x)$  are polynomials. Thus  $(3x - 11) - (2x + t) = x - 11 + t$  is divisible by  $x + 1$ . Hence  $11 + t = -1$ .

26. (c) The radius of  $B$  is half the radius of  $A$  so the length of the arc on  $B$  is  $\frac{\pi/4}{\pi/6}$  times  $x/2$ .

27. (b)  $983 = ad + r$ ,  $1183 = bd + r$ ,  $1333 = cd + r$  for some numbers  $a, b, c, r$ . Subtracting,  $200 = (b - a)d$  and  $150 = (c - b)d$ . Thus  $d$  divides both 150 and 200; the largest value for  $d$  is then 50. Note 50 gives a remainder of 33 for the given three numbers.

28. (c) If  $x$  is the number then  $1/x = x - 1$ . Solve for  $x$  by the quadratic formula and choose the positive root.

29. (d)  $a^b < 1 = (a + b)/2 < b^a < b < b/a$  narrows the possibilities to  $b/a$  or  $ab^2$ . If  $b = 1 + x$  and  $a = 1 - x$  where  $x = .001$  then  $ab^2 < b/a$  is equivalent to  $x + x^2 - x^3 > 0$  which is true.

30. (a) It is the product  $(2^3)(3^2)(5)(7)$ .

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