**David Essner Exam 14 1994-1995**

1. A class of students has 10 juniors and 20 seniors. If the average grade of the juniors is 80 and of the seniors is 92 then the class average grade is

(a) 85 (b) 86 (c) 87 (d) 88 (e) 89

2. If an equilateral triangle has length of side $x$ then the distance from each vertex to the intersection of the medians is

(a) $x/2$ (b) $2x/3$ (c) $x/3$ (d) $x/\sqrt{2}$ (e) $x/\sqrt{3}$

3. A school club has 40 girls and 30 boys. For a fund raiser the girls sold an average of $x$ boxes of doughnuts and the boys an average of $y$ boxes. If 70 percent of all boxes sold were sold by the girls then the ratio of $x$ to $y$ is

(a) 2 to 1 (b) 9 to 4 (c) 3 to 2 (d) 5 to 2 (e) 7 to 4

4. If the sum of the first 5 terms of an arithmetic progression is 90, and the sum of the 3 largest of these is 5 times the sum of the two smallest then the first term in the progression is

(a) 4 (b) 9/2 (c) 5 (d) 11/2 (e) 6

5. If $A,B,C$ are statements and it is assumed both ($I$) and ($II$):

($I$) Either $A$ is false or $B$ is true

($II$) Either $B$ or $C$, but not both, are true

then it may be concluded

(a) $A$ is true or B is false (b) Either $A$ or $C$ is true (c) Not both $A$ and $C$ are true

(d) $C$ is true and $A$ is false (e) Both $A$ and $C$ are false

6. The system of equations

$$\begin{aligned} x + y + 2z &= 2 \\ -x + 2z &= 2 \\ ax + by + cz &= d \end{aligned}$$

has a unique solution if the following is not 0:

(a) $a + 3b + c$ (b) $2a - 4b + c$ (c) $3a - 2b + 2c$ (d) $a - b + 2c$

(e) $-a + 3b + c$

7 $\dfrac{\log_3 10}{\log_3 5} =$

(a) $\log_2 3$ (b) $\log_3 2$ (c) $\log_3 5$ (d) $\log_{10} 5$ (e) $\log_5 10$

8. Jar $A$ has 25 pounds of $x$% solution and jar $B$ has 35 pounds of $(2x)$% solution. If 10 ounces from jar $A$ is mixed with 20 ounces from jar $B$ then the result has $y$% solution where $y/x =$

(a) 5/3 (b) 4/3 (c) 3/2 (d) 7/5 (e) 5/2

9. Given a square with sides of length 2, then the area between the inscribed and circumscribed circles is

(a) $4\pi$ (b) $2\pi$ (c) $4\pi/3$ (d) $3\pi/2$ (e) $\pi$

10. What amount $\$P$ when invested at 8% compounded 4 times per year will equal $1,000 in 20 years?

(a) $\frac{250}{1.08^{20}}$ (b) 400 (c) $\log_{10}\frac{1000}{1.16}$

(d) $\frac{1000}{1.02^{80}}$ (e) $\frac{1000}{1.6^4}$ $1000/(1.6)^4$

11. The expression $\log_{10}\frac{x-7}{\sqrt{9-x^2}}$ gives a real number for which values of x?

(a) $x < 7$ (b) $-3 < x < 3$ (c) $3 < x < 7$ (d) $-3 < x < 7$ (e) no values

12. If $\sin 2x = \frac{2\sqrt{2}}{3}$ then a possible value for $\sin^2 x$ is

(a) $\sqrt{3}/6$ (b) 1/3 (c) 2/9 (d) $\sqrt{2}/4$ (e) 3/8

13. John and Bill run a race. Bill runs at a constant speed. If John runs 9/10 as fast as Bill the first 2/3 distance of the race and then runs $x$ times as fast as Bill the remainder of the race then they will end in a tie if $x$ =

(a) 10/9 (b) 3/2 (c) 7/5 (d) 9/7 (e) 4/3

14. The line $y = x + 3$ is shifted 2 units to the right and then rotated 90 degrees counterclockwise about the origin. The equation of the resulting line is $y$ =

(a) $-2x - 3$ (b) $-x - 1$ (c) $-x + 5$ (d) $-2x - 5$ (e) $x - 5$

15. Bill has $64 and makes 6 bets, each time winning or losing half of his amount (= $64 + winnings - losses). If he wins 3 of the times and loses 3 of the times then he will have

(a) $64 (b) $32 (c) $27 (d) $96
(e) depends on the order of the wins and losses

16. Let $abc$ and $cba$ be two three digit numbers. If their product is 92,565 then $a + b + c$ =

(a) 6 (b) 8 (c) 10 (d) 12 (e) 14

17. The equation $x + 2y + 3z = 19$ has how many solutions $(x,y,z)$ where $x,y,z$ are all positive integers?

(a) 9 (b) 13 (c) 21 (d) 25 (e) 32

18. The quantity $(1001)^{1/3}$ - 10 is nearest the value

(a) 1/300 (b) 1/3000 (c) 3/100 (d) 3/1000 (e) 1/600

19. If $x$ is a large positive number then $\frac{x-1}{\sqrt{x-1}}$ is

(a) a large positive number (b) a large negative number (c) near 0
(d) near 1 (e) none of (a)-(d)

20. John makes a series of bets, each time winning or losing $1. The probability of winning each time is 1/2. He plans to quit if his net winnings (winnings - losses) is $5 or more. What is the probability he will quit after exactly 7 bets?

(a) 2/27 (b) 5/128 (c) 1/32 (d) 5/64 (e) 7/256

21. Given the equation $x^4 + ax^3 + bx^2 + cx + d = 0$ has two real double roots $r,s$ then $r + s =$

(a) $-a/2$ (b) $\frac{b+c}{2}$ (c) $a + b$ (d) $2c$ (e) $b - a$

22. The least common multiple of the integers 1 through 10 is

(a) 210 (b) 4200 (c) 2520 (d) 12,600 (e) 10!

23. If $2^{1000}$ is divided by 7 the remainder is

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

24. If $a = 0.99999$ and $b = 1.00001$ then which number is the largest?

(a) $ab$ (b) $a^2 b$ (c) $a/b$ (d) $ab^2$ (e) $b/a$

25. Let x(0) = 1/2 and $x(n) = 1/2 + \frac{x(n-1)}{4}$ for $n$ = 1,2,3,.... Then for $n$ very large $x(n)$ is nearest

(a) 1/2 (b) 2/3 (c) 3/4 (d) 4/5 (e) 5/6

26. How many digits are there in the number $3^{20}$ ?

(a) 8 (b) 9 (c) 10 (d) 11 (e) 12

27. Given a triangle whose sides are of length 3,4,5 then the radius of the circumscribed circle is

(a) $\sqrt{6}$ (b) $\sqrt{30}/2$ (d) 25/12 (d) $2\sqrt{2}$ (e) 5/2

28. Of 9 girls in a sorority John knows 8, Bill knows 7 and Tom knows 5. What is the least possible number of the girls known by all three?

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

29. Let $A$ and $B$ be sets and ' denote complement of a set. If $A \cap B'$ has 7 elements, $A' \cap B'$ has 4 elements and $(A \cap B)'$ has 12 elements then $A' \cap B$ has how many elements?

(a) 0 (b) 1 (c) 9 (d) 15 (e) 23

30. Three different numbers are selected at random from the set of integers 1 through 10. The probability that the smallest of the three integers is 4 is

(a) 1/8 (b) 3/32 (c) 5/64 (d) 11/120 (e) 2/5

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