. **Answers and Brief Solutions to E1996**

1. (d) .2x3 + .4x2 + .3x1 = 1.7

2. (e) 20 - 8 = 12 students take English; 12 - 4 = 8 students take both English and Math; 14 - 8 = 6 students take Math but not English.

3. (b) If x is the original length of a side then the change in volume is $(1.1x)^3 - x^3 = .331x^3$.

4. (d) Long division gives $8 - \frac{64}{8+x}$ which for $x > 0$ is an integer only if x = 8, 24, 56.

5. (e) Equate $4/3\ \pi r^3 = 4\pi r^2$ and solve for r.

6. (b) This makes the sum of the first 3 equations equal the fourth.

7. (a) Solve $2 = (1 + r)^n$ for n by taking the log of both sides.

8. (d) Let r be Bill's rate. Equating the times of John and Bill gives $\frac{D}{9r/10} + \frac{1-D}{11r/10} = \frac{1}{r}$. Solve for D.

9. (d) There are $C(10,3) = 120$ ways to choose 3 numbers from ten and $C(5,3) = 10$ ways to chose 3 odd numbers from these ten. Answer is 1 - 10/120.

10. (a) Let the terms be a, $a+d$, $a+2d$, ... Then $a = 3$ and $10a + 45 = 120$. Thus $d = 2$ and the terms are 3,5,7,9,...

11. (e) Let the altitude from C intersect AB in point P. If x is the length of AP and y the length of BP then $2/3 = 6/x$ and $3/4 = 6/y$. Thus the length of AB is $x + y = 9 + 8 =$ 17 and the area is (1/2)x6x17 = 51.

12. (c) Solve $10(.1) + 20(.2) + x(.3) = (30 + x)(.25)$ for x.

13. (a) Let (a,b) be the point of intersection. Then $\frac{b-2}{a-1} = -\frac{a}{b}$ gives $a^2 + b^2 =$ $a + 2b = 1$. Solving gives $b = 4/5$ and $a = -3/5$ as a solution.

14. (b) Solve simultaneously $b = 5a$ and $\frac{b-3}{a-2} = -1/5$.

15. (c) Long division of $x^3 + ax + bx + c$ by $x - 1$ gives a remainder of $a + b + c + 1$.

16. (d) Multiply the numerator and denominator by $\sqrt{1+x}$ + 1 and simplify; then substitute $x = 0$.

17. (c) Use the binomial approximation $(x^2 + 1) = (x^2)^{1/2}$ +
$(1/2)(x^2)^{-1/2}$ + $(1/2)(1/2)(-1/2)(x^2)^{-3/2}$ + ... and simplify.

18. (d) The second person has probability 6/7 of a different birth weekday than the first person, and the third person a probability of 5/7 different from the first two. Form the product (6/7)x(5/7).

19. (c) Multiply both sides of the equation by $x^2(x + 1)$ and equate the constant terms and coefficients of x , x^2 on the two sides of the equation. Then $A + C = 1$, $A + B = 9$, $B = 7$. Solving gives $A = 2$, $B = 7$, $C = -1$.

20. (b) Consider a triangle with vertices O, the center of the circles, P the point of tangency, and Q a point of intersection of the tangent with the larger circle. The lengths of the sides are 1 for OP, 2 for OQ and hence $\sqrt{3}$ for PQ.

21. (b) From $360 = 2^3 \text{x} 3^2 \text{x} 5$ and $756 = 2^2 \text{x} 3^3 \text{x} 7$ the least common denominator is $2^3 \text{x} 3^3 \text{x} 5 \text{x} 7 = 7560$. The numerator is then 3x7x7 + 2x5x5 = 197.

22. (e) Take log of both sides of the equation. This gives $y^2 = 2 + y$ where $y = \log_a x$. Solving gives $y = -1,2$ and hence $x = 1/a$ and a^2 .

23. (c) $x(1) = 1 + .1$, $x(2) = 1 + .1 + .1^2$, ... Thus $x(n)$ is a geometric progression with initial term 1 and ratio .1; the limit value is $\frac{1}{1-.1}$ = 10/9.

24. (a) Note $3^{20} = (3^4)^{25}$. The remainder of 3^4 divided by 11 is 4 and the remainder of 4^5 by 11 is 1.

25. (e) A is false if y is odd and B is false is z is even ; C is true since yz must be even and the sum of an even and odd number is odd.

26. (e) Note $1000 = 2^3 \text{x} 5^3$. The 8 divisors of 1000 less than $\sqrt{1000}$ (< 34) are 1,2,4,8,5,25,10,20.

27. (d) Let p be the probability the person with \$2 will win the \$3; then 1 - p is the probability the person with \$1 will win the \$3. Therefore $p = 1/3 + 2/3\,(1 - p)$; solving gives $p = 3/5$.

28. (b) If $0 < x < 1$ and $a < b$ then $x^b < x^a$. Therefore $x = x^1 < x^x = y < 1$ and $z = x^y < x^x = y$. Also $x = x^1 < x^y = z$.

29. (c) $1 + i = \sqrt{2}\,(\cos \pi/4 + i \sin \pi/4)$ and by DeMoivre's Theorem
$(1 + i)^{10} = \sqrt{2}^{\,10}(\cos 10\pi/4 + i \sin 10\pi/4) = 32i$.

30. (a) If c is the third side then the altitude to c has length $10 \sin 30° = 5$; there are two triangles if x is greater than this value and less than the length of b.