Answers and Brief Solutions to E1998

1. (a) If x is the score on the final then $(90 + 80 + 90 + 60 + 2x)/6 = 85$.

2. (a) The final price is $(1.1)*(.9) = .99$ the original price.

3. (b) The probability the second differs from the first is 5/6, and in this case that the third differs from the first two is 2/3. Answer is 5/6x2/3.

4 (e) From $2x + 5 = 3x - 2$ and $2x + 5 = -(3x - 2)$ the two solutions are 7 and -3/5.

5. (c) = 5x5 = 25 since each number is a product of a power of 2 and a power of 3, each power between 0 and 4 inclusive.

5 (d) This is the contrapositive of the given expression.

7. (a) If n denotes the number of elements in a set then $n(A\cap B) = n(A\cup B) - n(A'\cap B) - n(A\cap B')$; this can be easily seen from a Venn diagram.

8 (c) If x is the length of the side then the volume is x^3 and the total surface area is $6x^2$.

9. (e) $x(n) = a/2^n - (1 + 1/2 + 1/4 + ... + 1/2^{n-1}) = a/2^n - 2 + (1/2)^{n-1}$. Thus $0 = a/64 - 2 + 1/32$ gives $a = 126$

10. (d) Let D be the endpoint of the altitude. Then $AD = \sqrt{3}\,h$ and $DB = h/\sqrt{3}$. Then $AB = AD + DB = 4h/\sqrt{3}$ and $10 = 1/2\ h(AB)$ gives the result.

11. (e) The value is $(1.1)^{100}$. Since $1.1^2 = 1.21$ and $1.1^4 = 1.21^2 > 1.46 > \sqrt{2}$ then $1.1^8 > 2$ and from $1.1^{100} = 1.1^{96} \times 1.1^4$ then $1.1^{100} > 2^{12}\sqrt{2} > 4{,}000\sqrt{2}$; the actual value is near 13,780.

12. (b) Equating the distances from (a,b) to the three points and solving for a and b gives $a = 1/4$ and $b = 5/4$.

13. (d) Let j and b be the speeds of John and Bill. Then $j + 7 = 2b$ and $b = 4/5\,j$. Thus $j + 7 = 8/5\,j$.

14. (b) Simplify $.03x + .06y + 10(.05) = .04(x + y + 10)$

15. (a) Divide $x^3 - 7x + 6$ by $(x - 1)(x - 2)$ to obtain $x + 3$.

16. (d) $= \dfrac{2-x}{2x(2-x)} = \dfrac{1}{2x}$ if $x \neq 2$.

17. (e) = 6/7 - 5/6 (= $s(6) - s(5)$ where $s(n) = a(1) + a(2) + ... + a(n)$).

18. (d) The angle subtended by the arc is $\pi/3$ radians, which is 1/6 of a complete revolution. The area of the cone shaped region is then 1/6 the area of the circle.

19. (a) Let x be the length of the bridge and y the distance from the train to the far end of the bridge. Then $.6x/r = y/50$ and $.4x/r = (y - x)/50$. Solve for r.

20. (c) The intersection of the inequalities geometrically describes a triangle with vertices (-2,0), (1/2,5/2) and (6/7,10/7).

21. (b) The values $y_1, y_2, ..., y_{10}$ are respectively the base x raised to the exponent 2, 2^2, 2^3, ...,2^{10} and $2^{10} = 1{,}024$.

22. (e) It is $C(12,3)\text{x}2^3$ = (12x11x10)/(3x2x1) x 8 where C(12,3) is the binomial coefficient.

23. (a) Let $x = 5 + 6m$ and $y = 2 + 3n$ where m,n are integers. Then $xy - 10 = 15n + 12m + 18mn$ and of the numbers 2,3,6 only 3 divides each of 15, 12, 18.

24. (b) The ones with 1 the smallest integer are (1,2,17),(1,3,16),...(1,9,10) giving 8; similarly with 2 obtain (2,3,15),...,(2,8,10) giving 6. Continuing the answer is 8+6+5+3+2 = 24.

25. (c) Let r be the ratio of each term to the preceding. Then $r^2 + r = 10/9$ gives $r = 2/3$. Thus the difference is 2/3 - 4/9.

26. (c) sin B = 3/5 and by the law of sines, $\dfrac{10}{\sin 120} = \dfrac{b}{3/5}$.

27. (e) Substitute $y = 1 - x$ into $y + z = -1$ and solve simultaneously with $x + z = 4$ to get $z_0 = 1$ and $x_0 = 3$. From $w + x + y = 6$ the sum is $6 + z_o = 7$. .

28. (b) 1997/97 = 20 with remainder 57 and 57/19 = 3; thus $m = 3$, $n = 20$ is a solution. It is the only one since if $1997 = 19(3 + a) + 97(20 + b)$ then $19a + 97b = 0$ and this is only possible if a is a negative multiple of 97 or b a negative multiple of 19 and these do not give other valid answers.

29. (d) The numerator is defined for $x \le 1$, and the denominator for $x \ge 0$; if $x = 1$ the denominator is 0 and the fraction is not defined.

30. (c) There are $C(10,3)$ = (10x9x8)/(3x2x1) = 120 combinations of 3 balls from 10 and 8 of these have two black balls; answer is 8/120 = 1/15.