Answers and Brief Solutions to E1999

1(c) The sequence is 2,3,5,8,13,21,34,55,89,144

2(d) = 1 + 3 + 5 + ... + 37 = (1+2+...+37)-2(1+2+...+18) = 37x38/2 -2x18x19/2 = 19x(37 - 18) = 19^2 or the points form a right triangle with sides of length 19 and 38 giving an area 1/2x19x38.

3(b) If ab were rational then $ab = m/n$ where m,n are integers and $b = m/an$ would then be rational.

4(e) $(x - y)(x + y) = x^2 - y^2 = x - y$ gives $x + y = 1$.

5(c) If D is the distance between the towns and t the time until they meet then $D = Dt/9 + Dt/6$; solve for t.

6(a) $z/x = x/y$ gives $2/1 = 1/y$ and hence $y = 1/2$. Thus if $z = 4$ then $x^2 = 4(1/2) = 2$.

7(c) The sum of the remaining 8 scores is 10x76 - 46 - 52 = 662 and 662/8 is nearest 83.

8(c) If r is John's rate and t is the time sought then $r(t + 5) = (4r/3)t$; solve for t.

9(b) If $x \leq -1$ the expression is $2 - x - x^2$ which has maximum value 2 at $x = -1$; if $-1 \leq x \leq 1$ then the expression is $(x - 1/2)^2 - 1/4$ which also has maximum at $x = -1$, and if $x \geq 1$ then the expression is $1/4 - (x - 1/2)^2$ which has maximum 0 at $x = 1$.

10(c) = (20)(21)/2 - (21)(7) = 63

11(b) If (a,b) is the center then $a^2 + b^2 = a^2 + (b-2)^2 = (a - 1)^2 + (b - 3)^2$
Solving the first two gives $b = 1$; substituting this value in the first and third and solving for a gives $a = 2$.

12(a) From $(x - a)(x - b)(x - c) = x^3 + x^2(-a - b - c) + x(ab + ac + bc) + (-abc)$ the sum of the roots is -(coefficent of x^2)and the product is -(constant term).

13(e) If taxed the person retains (100 - 20)% = 80% of the interest and 80% of 8% is 6.4%.

14(e) $8^p = 3$ gives $2^{3p} = 3$ and hence $(3p)\log_3 2 = \log_3 3 = 1$.

15(b) $3^x = 7^y$ and $7^y = 10$ gives $3^{xy} = (3^x)^y = 7^y = 10$

16(c) = $f(4 - 5) = f(-1) = -3 + 1$

17(d) In the interval [0,1] those numbers which are at least twice as far from one end of [0,1] as the other are the numbers in the intervals [0,1/3] or [2/3,1].

18(c) $f(g(x)) = 3(ax + b) + 2 = 15x + 8$; thus $3a = 15$ and $3b = 6$.

19(a) It is $C(9,6)(2^6) = 84 \text{x} 64$ where $C(9,6)$ is the binomial coefficient

20(b) By deMoivre's theorem it is $(\sqrt{2})^{10}$ (cos (-10π/4) + i sin(-10π/4))

21. (c) The equation $.1x + .3y = .15(x + y)$ gives $.15y = .05x$.

22(d) Let $y = 3x + b$ be the equation of the line; then $6 = (3)(3) + b$ gives $b = -3$.
Find the simultaneous solution of $y = 3x - 3$ and $y = 3/2\, x^2$

23(b) Since -1 is a root of $x^{15} + 1$ it follows that $x - (-1) = x + 1$ is a divisor.

24 (c) Solving $\frac{2x+4}{x} = \frac{5x+10}{2x+4}$ = 5/2 gives x = 8 and ratio 5/2. Thus the sequence is 8,20,50,125

25(e) If x is the length of the sides of S then by similar triangles $(3 - x)/x = 3/4$.

26(d) The prime factorization of 630 is 2x3x3x5x7; thus if $630N$ is a perfect square it must include the square of each prime factor. Thus 2x5x7 = 70 gives the smallest possible value for N.

27(a) Subtract equation (2) from equation (1) to obtain $2x = 4$, or $x = 2$. Add equations (1) and (3) to get $2u + 2x = 6$ or $u = 1$. Equations (1),(4) are now $y + z = 2$ and $-y + z = 4$. Adding gives $2z = 6$ or $z = 3$ and $y = -1$. Answer is 1 - 2 - 1 - 3 = -5.

28(e) $x = 8 + 2t$, $y = 1 - 3t$ is a solution for any integer t.

29(e) Letting $C(n,r)$ denote the number of combinations of r objects from n then the answer is $\frac{C(3,2)C(7,1)}{C(10,3)}$.

30(d) $(x + 1)^2 - y^2 = 1 - a$. If $a = 1$ then the graph consists of two straight lines.