Answers and Brief Solutions to E2000

1. (e) From (60x11 + 70x19)/30 = 66 1/3

2. (c) is not betwen $-\pi/2$ and $\pi/2$

3. (d) It travels 5 times around a circle with perimeter $2\pi x3 = 6\pi$

4. (c) From 60% of 30 = 18 and 50% of 90 = 45 they must win 45-18 = 27 of 60 games and 27/60 = .45.

5. (e) The equation $x + 2x = 1$ has solution $x = 1/3$; the equation $-x + 2x = 1$ has solution $x = 1$ but 1 does not satisfy the given equation.

6. (c) If P and Q are statements then the negation of (P or Q) is equivalent to (not P and not Q)

7. (d) $\dfrac{x^3-1}{x-1} = x^2 + x + 1$ if $x \neq 1$ and $1^2 + 1 + 1 = 3$

8. (b) 5,12,13 are the values.

9. (a) Dividing $2x^3 - x^2 - x - 3$ by $x^2 + x + 1$ gives $2x - 3$ which has root $x = 3/2$.

10.(b) The identity is 0 since $a*0 = 0*a = a$. From 3*(-3) = (-3)*3 = 0 the result follows.

11.(e) If s is the amount of salt and w is the final weight then $s = .01x = .02w$ so $w = .5x$

12.(a) Let A,B,C,D be assigned $1,b,c,d$. Then $1 = \dfrac{b+d}{2} = c$ and $b = \dfrac{a+c}{2} = d$ gives $a = b = c = d = 1$.

13.(e) Note $x_{n+1} = 2x_n - x_{n-1}$. The sequence for terms 0,1,2,3, is 1,4,7,10,... . with general term $x_n = 3n + 1$; thus $x_{100} = 3(100) + 1$.

14. (b) The probability the box with three balls has different colors is (1x2x3)/C(6,3) = 3/10. Assuming this the probability the box with two balls has different colors is (1x2)/C(3,2) = 2/3. The answer is (3/10)x(2/3) = 1/5.

15.(c) If r_t is the speed of Tom and r_b is the speed of Bill then $100/90 = r_t/r_b = (100 + x)/100$. Solve for x.

16.(d) Letting $C(m,n)$ denote the binomial coefficient then in the expansion one may select the a's from $C(8,3)$ terms, then the b's from $C(5,2)$ terms and then the c's from $C(3,3)$ terms. The answer is $C(8,3)C(5,2)C(3,3)2^2 = 56 \text{x} 10 \text{x} 1 \text{x} 4$.

17.(b) By the quadratic formula the roots are $3 + 2\sqrt{2}$ and $3 - 2\sqrt{2}$

18.(c) Write the circle in the form $(x - 3)^2 + (y - 2)^2 = 1$. Then consider the triangle with vertices P, (3,2) and Q with right angle at Q. By the Pythagorean Theorem $d^2 = [(3-1)^2 + (2-1)^2] - 1^2 = 4$.

19.(d) $z = 2(\cos(300°) + \sin(300°)i)$ and by Demoivres Theorem $\sqrt{z} = \sqrt{2}(\cos(150°) + \sin(150°)\, i) = -\sqrt{6}/2 + \sqrt{2}/2\, i$.

20.(e) If $x > 1$ the inequality becomes $x + 1 > x - 1$ which is true for all x; if $x < 1$ then the inequality becomes $x + 1 > 1 - x$ which is true for $x > 0$.

21 (b) $2 \log x = \log x^2$ and $x^2 = 2x$ gives $x = 0$ and $x = 1$. However log 0 is not defined so $x = 1$ is the only solution.

22.(a) If O is the center of the circle then OP has length $r - 1$ and is a leg of a right triangle with the other leg of length $4/2 = 2$ and hypotenuse of length r. Thus $r^2 = 2^2 + (1 - r)^2$; solve for r.

23.(b) During the four years P is multiplied by $(1.2)^2(.9)^2 = 1.166..$

24. (a) From the equations $c - b = b - 8$ and $c/b = b/9$ it follows that $c = 2b - 8$ and $b^2 = 9c$.
Combining gives $b^2 - 18b - 72 = (b - 12)(b - 6) = 0$. Then $b = 12$ since the sequence is increasing.

25.(d) $x^2 - xz + yz - y^2 = x^2 - y^2 - xz + yz = (x + y)(x - y) - z(x - y) = (x + y - z)(x - y)$

26 (e) $x - y = 2m$ and $y - z = 3n$ for some integers m,n give $x - z = 2m + 3n$ which is not necessarily a multiple of 3, 6, or 8. For example if $x = 8$, $y = 6$, $z = 3$ then none of *I, II, III* are true.

27.(c) $10^{20}/15^{15} = (10^4/15^3)^5 = [(2/3)^3(10)]^5 > 1$ and $15^{15}/20^{10} = (15^3/20^2)^5 = [(3/4)^2(15)]^5 > 1$.

28.(e) 1-9 gives 9; 10-99 gives 90x2 = 180; 100-999 gives 900x3 = 2700; 1000 gives 4

29 (d) By algebra obtain the quadratic equation $y^2 - xy - x^2 = 0$. Solve for y using the quadratic formula and choose the positive value.

30.(a) Let $40 - \sqrt{x}$ assume the values 1,4,9,16,25,36 and get $x = 39^2, 36^2, 31^2, 24^2, 15^2, 4^2$.