

Answers and Brief Solutions to E2006

1. (c) Let $C = \pi D$ where D is the diameter. Then solving $C + 1 = \pi(D + x)$ simultaneously with $C = \pi D$ for x gives $x = 1/\pi$.
2. (e) Let BD be the altitude from B to AC ; then $|BD| = 2\sin 60^\circ = \sqrt{3}$. Also $|AD| = 2\cos 60^\circ = 1$ and hence $|DC| = 3 - 1 = 2$. Thus $|BC|^2 = (\sqrt{3})^2 + 2^2 = 7$
3. (b) The terms of the sequence are $a, b, a+b, a+2b, 2a+3b, 3a+5b, 5a+8b$. Solving $3a + 5b = 21$ and $5a + 8b = 34$ simultaneously gives $a = 2, b = 3$.
4. (d) From $(3 + x)/2 = -1$ and $(-2 + y)/2 = 5$ it follows that $x = -5$ and $y = 12$.
5. (b) Initially there were the four equally likely possibilities RR, RG, GR, GG . If at least one is red then there are the three equally likely cases RR, RG , and GR and only one of these has two red balls.
6. (a) Using the quadratic formula the sum is $-b$.
7. (d) Let x be the unknown; then $(2 + x)/(10 + x) = 0.4$. Solve for x .
8. (d) The student must get more than $(.9)(75) - 17 = 50.5$ correct answers on the remainder of the test
9. (b) Solve $x = 20 + y$ and $x = 1.1y$ to get $x = 220$.
- (a) 1 (b) $3/2$ (c) 2 (d) $2\sqrt{2}$ (d) $3\sqrt{2}$
10. (c) The region is the diamond shaped figure with vertices $(1,0), (0,1), (-1,0), (0,-1)$ which can be partitioned into 4 triangles each with area $1/2$.
11. (e) There are $9 \cdot 1 = 9$ digits from 1 to 9, $90 \cdot 2 = 180$ digits from 10 to 99; $900 \cdot 3 = 2700$ digits from 100 to 999 and 4 digits for 1,000. The answer is $9 + 180 + 2700 + 4$.
12. (b) Let $x = 10a + b$. Then $2(10a + b) = 10b + a + 14$. Hence $19a - 8b = 14$, and this has the solution $a = 2, b = 3$.
13. (a) There are $92 - 48 = 44$ students who take math but not science and $56 - 48 = 8$ students who take science but not math; the sum is 52.

14. (d) Let there be x pennies and y dimes. Then $x + 10/3 x + 10 y = 360$; thus $13x + 30y = 1080$. Then $y = 36 - 13x/30$ so x is divisible by 30 and $13x < 1080$. The possible values of x are 30 or 60, and hence 60 pennies, 40 nickels and 10 dimes gives 110 as the answer.

15. (c) Solve $2P = P(1 + r/4)^{32}$ for r .

16. (d) $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x = 1 + \sin 2x = (5/4)^2$ gives $\sin 2x = 9/16$.

17. (a) $\log_4 a = 1/2 \log_2 a$ and $\log_8 b = 1/3 \log_2 b$. Thus $1/2 \log_2 a + \log_2 b = 1$ and hence $\log_2 ab^2 = 2$ from which $ab^2 = 4$

18. (d) If there are B boys and G girls and there are N questions on the test then the total number of correct answers is

$$[3/4 B + 2/3 G]N = [7/10 (B + G)]N.$$

Simplifying gives $1/20 B = 1/30 G$ from which $B/G = 2/3$.

19. (c) From $835 = 7 \cdot 119 + 2$ it follows that n can be any odd integer from 1 to 119 inclusive. There are $(1 + 119)/2 = 60$ such integers.

20. (b) If a is a real number then $|a - 1| > |a + 1|$ if and only if $a < 0$. Thus the solution set of the given inequality is the solution set of $x^2 - 4x = x(x - 4) < 0$ which is the interval $(0, 4)$.

21. (b) From $x^2 - y^2 = (x + y)(x - y) = 403 = (31)(13) = (403)(1)$ it follows that $x + y = 31$ and $x - y = 13$ which gives $y = 9$ or $x + y = 403$ and $x - y = 1$ which gives $y = 201$.

22. (e) Form a triangle whose vertices are the center of the circle and two of the vertices of the given equilateral triangle. This triangle has two sides of length r , with included angle 120° , and the other side of length 2. By the Law of Cosines, $2^2 = 2r^2 (1 - \cos 120^\circ)$ which simplifies to $r^2 = 4/3$.

23. (e) Let $x = 1110$. Then $a = x/(x + 1)$, $b = (2x + 1)/(2x + 3)$ and $c = (3x + 1)/(3x + 4)$. Then $(2x + 1)(3x + 4) > (2x + 3)(3x + 1)$ implies $b > c$ and $(3x + 1)(x + 1) > x(3x + 4)$ implies $c > a$.

24. (c) $2^{24} - 1 = (2^6 - 1)(2^6 + 1)(2^{12} + 1) = 63 \cdot 65 \cdot 4097$. An even number cannot be a divisor and by long division arithmetic 61, 67 and 69 do not divide 4097.

25. (a) This sum is the set of all products in the expansion of
 $(1+2+3+4)(5+7+8)(9+10+11) = 10 \cdot 20 \cdot 30$

26. (a) The discriminant of $x^2 - 2x + 2$ is negative so it gives no points and of $x^2 + 2x - 2$ is positive so it gives two points; $x^3 + 1$ has one real root so the total is $2 + 1 = 3$.

27. (b) Let $y = 3^{2x}$; then $y + y^2 = 20$ gives $y = 4$. Then $3^{3x} = 4^{3/2}$

28. (e) Let p be the probability that Bill wins. Then $p = 1/6 + (5/6)(5/6)p$

29. (c) Since the equation has four prime number solutions, each must divide the number 1430 which is the product of 2,5,11 and 13; hence these are the only possible positive integer solutions

30. (e) Let x be the unknown, t the time for Tom to get from home to Mary's work and t' the time for Tom to reach Mary when she walked. Then Mary's travel times yield the equation $(x + t') + 40 = t + 60$ and Tom's travel times yield $2t = 2t' + 40$. Solving simultaneously gives $x = 40$.