Resource Theft and Spatial Population Dynamics

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(with Chris Cosner, Miami)

Resource theft (I)

Lion chases prey ... lion catches prey

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Lion chases prey ... lion catches prey

Lion fends off hyenas ... hyena steals some food

Resource theft (II)

Cheetah chases prey ... cheetah catches prey

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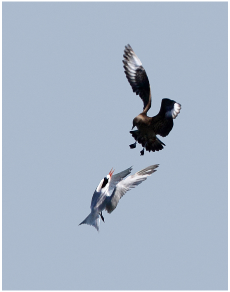
Cheetah chases prey ... cheetah catches prey

Lion chases cheetah off ... lion steals entire catch

Resource theft (III)



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- → A small number of scroungers can do well (many producers)
- → Many scroungers may do poorly (few producers)

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- $\rightarrow$ What if s depends on p in some way?

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$\rightarrow$ What is the behavior of the dynamical system?

Equilibrium states

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$\times$ $E_3 = (0, s^+)$ (scrounger only)

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→ If $mk > ce$ then

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- E_2 exists

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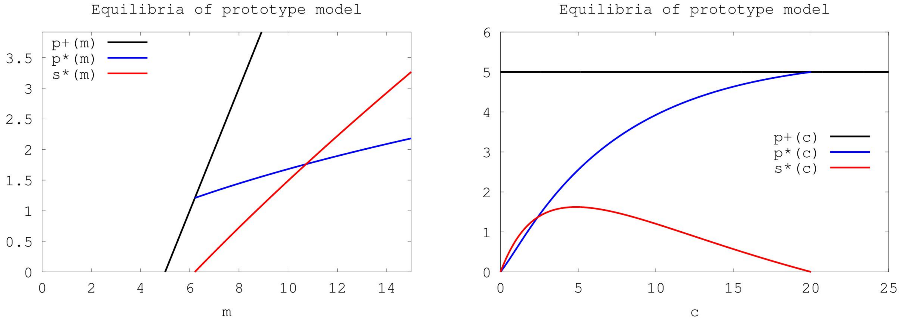
- E_2 exists
- E_2 is globally asymptotically stable (Lyapunov function)

Role of resource (m) and monopolization (c)



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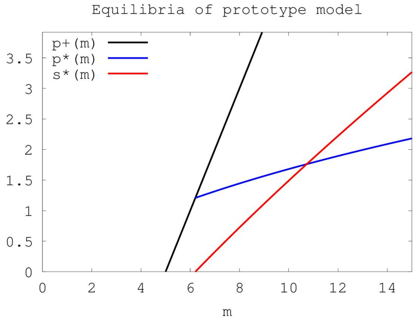

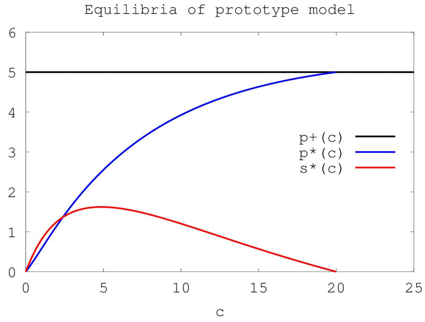

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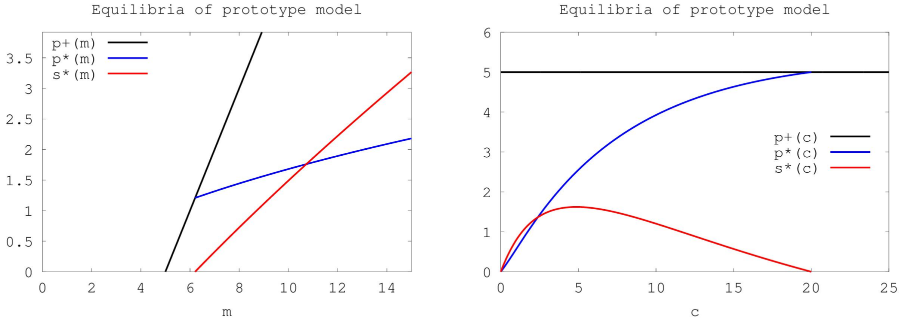

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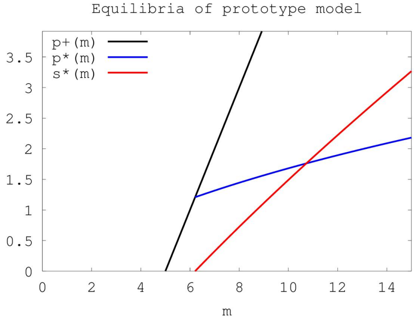

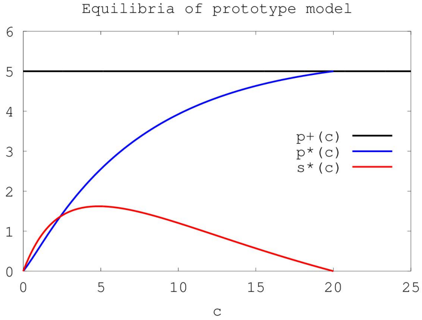

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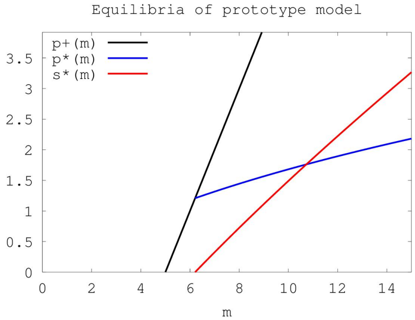

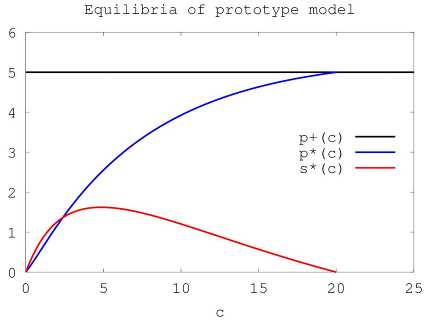

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What is the role of space?

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→ no-flux boundary conditions ($\partial_\nu p = \partial_\nu s = 0$ on $\partial\Omega$)

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→ $m(x)$: producer resource discovery rate (spatial profile)

→ d_1 and d_2: mobilities

→ no-flux boundary conditions ($\partial_\nu p = \partial_\nu s = 0$ on $\partial\Omega$)

→ how do the resource $m(x)$ and movement (d_1, d_2) combine to influence the ecological outcome?

Steady-states

A steady-state (p^*, s^*) satisfies

$$d_1 \Delta p^* + \left[\phi(s^*)m(x) - d - ap^*\right]p^* = 0, \qquad x \in \Omega$$

$$d_2 \Delta s^* + \left[\psi(s^*)m(x)p^* - e - bs^*\right]s^* = 0, \quad x \in \Omega$$

with no-flux boundary conditions

$$\partial_\nu p^* = \partial_\nu s^* = 0, \quad x \in \partial\Omega$$

and is non-negative everywhere

$$p^*(x) \geq 0 \quad \text{and} \quad s^*(x) \geq 0, \quad x \in \Omega$$

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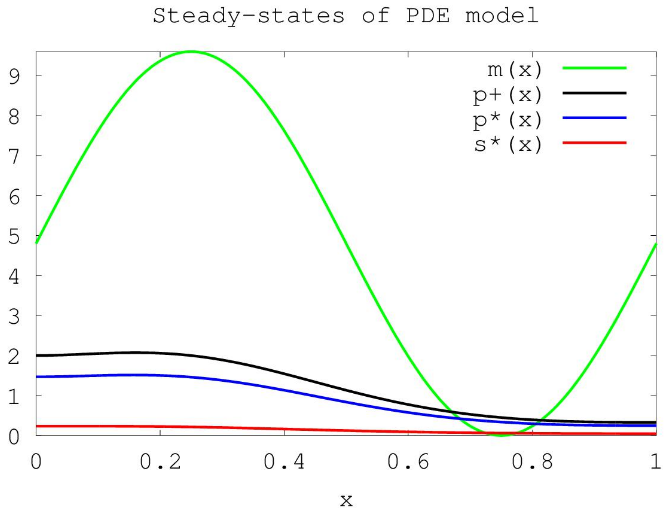
Profiles of E_1 and E_2



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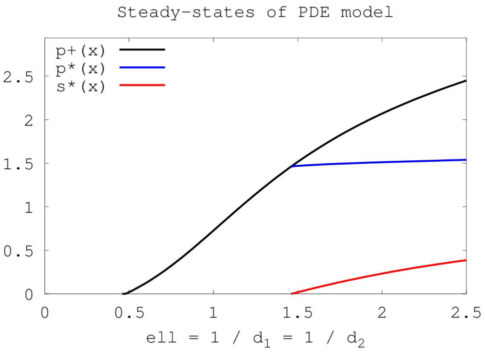
If $m(x)$ is constant then E_2 is unique and globally asymptotically stable

Slower dispersal is favored



Suppose d_1 and d_2 are replaced by d_1/ℓ and d_2/ℓ

The horizontal axis is ℓ

The vertical axis is the $L^\infty(\Omega)$-norm of the steady-states E_1 and E_2

Conditional movement

Allow producers and scroungers to move in response to the resource and/or population densities

$$\frac{\partial p}{\partial t} = \nabla \cdot \big[d_1 \nabla p - \beta_1 p \nabla f\big] + \big[\phi(s) m - d - ap\big] p, \quad x \in \Omega$$
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→ β_1, β_2 constants (sign affects interpretation)

→ $f = f(m, p, s)$ and $g = g(m, p, s)$

Strategies (f and g)

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0	random diffuser
m or $\ln m$	resource
p or $\ln p$	producer density
s or $\ln s$	scrounger density
$\phi(s)$ or $\psi(s)$	producer or scrounger share
$\phi(s)m$	producer resource acquisition rate
mp	corporate resource discovery rate
$\psi(s)mp$	scrounger resource acquisition rate
$\phi(s)m - d - ap$	producer fitness
$\psi(s)mp - e - bs$	scrounger fitness

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Happy Birthday Chris!