

## Main lectures.

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Title: The State of The Art in Numerical Relativity

Abstract: I will give an overview of recent developments and progress in numerical relativity. For the most part I will restrict attention to 3+1 approaches, and computations in 3 space dimensions plus time, with a particular focus on efforts to treat collisions of compact objects (black holes and neutron stars).

In terms of continuum aspects, topics to be discussed include formulations of the Einstein equations that show promise for stable, long term simulation of compact systems, choices of coordinate systems, the role of the constraints and constraint preservation, black hole (singularity) excision techniques, and boundary conditions. Topics related to the discretization of the equations of motion and the computational solution of the discrete equations will include a brief survey of advances in the application of finite difference and spectral techniques to problems in numerical relativity. Strategies for efficiently dealing with the multi-scale nature of many of the most pressing problems will be outlined.

I will also summarize the state of various group calculations to simulate the evolution of single black holes, binary black holes and binary neutron stars. Finally, I will also discuss recent developments in critical gravitational collapse, and related phenomenology arising in the study of nonlinear wave equations.

Throughout my lectures I will try to highlight some of the key unresolved issues that need to be successfully addressed before numerical relativity achieves its full potential as a means to compute generic solutions of Einstein's equations.

Thibault Damour  
IHES

Title: Cosmological Singularities, Einstein Billiards and Lorentzian Kac-Moody Algebras

Abstract: We review recent work on the general solution of (bosonic) Einstein-matter systems in the vicinity of a cosmological, *i.e.* spacelike, singularity. The asymptotic behaviour, as one approaches the singularity, of the general solution is found to be describable, at each (generic) spatial point, as a billiard motion in an auxiliary Lorentzian space. For certain Einstein-matter systems, notably for pure Einstein (Ricci = 0) in any spacetime dimension  $D$  and for the particular Einstein-matter systems arising in String theory, the billiard tables describing asymptotic cosmological behaviour are found to be identical to the Weyl chambers of some Lorentzian Kac-Moody algebras. In the case of the bosonic sector of supergravity in 11 dimensional spacetime the underlying Lorentzian algebra is that of the hyperbolic Kac-Moody group  $E_{10}$ , and there exists some evidence of a correspondence between the general solution of the Einstein-three-form system and a null geodesic in the infinite dimensional coset space  $E_{10}/K(E_{10})$ , where  $K(E_{10})$  is the maximal compact subgroup of  $E_{10}$ .

George F.R. Ellis  
 University of Cape Town  
 Lecture 1

Title: The space of cosmological space-times

Abstract: I first focus on how to best describe the space of cosmological space-times and what its essential properties are and some comments on the dynamical behaviour revealed by studies that will be described in more detail by John Wainwright. I will then relate this both to observations and to anthropic issues [i.e. the possible existence of observers]. This space includes some viable singularity free solutions which will be briefly described, thus posing the issue of the tension between very special initial conditions and the existence of initial singularities. I will conclude with remarks on the issue of realised infinities in this context and the concept of multiverses

Lecture 2

Title: Dynamical Properties of cosmological solutions

Abstract: I will consider domains of dependence in theory and in practice, showing how while there are nice FOSH formulations and associated results in relativistic cosmology, the characteristics of scalar and vector perturbations are timelike, hence the real domains of dependence in cosmology are much smaller than implied by the light cone. This can be clearly seen in relation to the different physical conditions at different stages of evolution of the universe. I will briefly relate this to the issue of the arrow of time and how that problem looks different at each of these epochs. Finally I will give some brief comments about the nature of physically important gravitational effects at these different stages and put the question: are there any epochs in the evolution of realistic universe models where tensor (gravitational wave) modes are important? This will be distinguished from the Bianchi Type IX oscillations which are 'silent universes' characterised by ODE's. The issue here is what characterises a gravitational wave; I suggest that the criterion is that both  $\text{curl}E \neq 0$  and  $\text{curl}H \neq 0$ .

Gerhard Huisken  
 AEI

Lecture 1

Title: Surgery for Geometric evolution Equations

Abstract: The lecture explains and compares common and distinctive features in recent surgery results for mean curvature flow and Ricci flow.

Lecture 2

Title: A priori estimates for mean curvature flow of 2-convex surfaces

Abstract: The lecture explains joint work with Carlo Sinestrari on a priori estimates for curvature and gradient of curvature in mean curvature flow of surfaces having the sum of the smallest two principal curvatures positive.

Lecture 3

Title: Surgery in mean curvature flow of 2-convex surfaces

Abstract: The lecture explains the actual surgery construction near singularities of 2-convex surfaces moving by mean curvature.

Sergiu Klainerman  
 Princeton

Title: On the Cauchy problem in GR I

Igor Rodnianski  
 Princeton  
 Title: On the Cauchy problem in GR II, III

Rick Schoen  
 Stanford  
 Titles:

- 1) The constraint equations I
- 2) The constraint equations II
- 3) The Jang equation, black holes, and energy

Abstract: In this series of three lectures we will discuss the constraint equations and their geometric significance. For asymptotically flat solutions, we will introduce natural asymptotic conditions which are especially well adapted to discussion of the asymptotic conserved quantities of relativity. We will introduce localized solution techniques and give applications to constructions of solutions with optimally simple asymptotic behavior. We will also use closely related techniques to construct perturbations of a given solution of the vacuum constraint equations with specified angular momentum. Finally we will give a discussion of the Jang equation and its connection to black holes and gravitational energy.

Daniel Tataru  
 UC Berkeley  
 Title: Quasilinear wave equations

Gang Tian  
 Princeton

Title: Curvature estimates for certain geometric equations, I & II.

Abstract: In this talk, I will discuss new basic curvature estimates for Einstein metrics and the Ricci flow. These include my recent work with J. Cheeger on Einstein manifolds, as well as new curvature estimates of G. Perelman for the Ricci flow. I will also discuss some applications of these estimates to geometric problems.

John Wainwright  
 University of Waterloo

Title: Cosmological models from a dynamical systems perspective

Abstract: It is useful to study the space of all cosmological models from a dynamical systems perspective, that is, by formulating the Einstein field equations as a dynamical system using appropriately normalized variables. I will discuss various aspects of this work, the choices of normalization factor, multiple representations of models, the past attractor, nonlinear dynamics in close-to-Friedmann-Lemaitre models, Weyl curvature dominance, and numerical simulations.

### **Mathematical GR Session.**

Mike Anderson  
 SUNY Stonybrook

Title: Relations between deSitter-type space-times and Riemannian AdS spaces.

Niky Kamran  
 McGill University

Title: The local contact geometry of differential equations and conformal Lorentzian structures in dimensions three and four.

Alan Rendall  
AEI

Title: Asymptotics of solutions of the Einstein equations with positive cosmological constant

Hans Ringström  
AEI

Title: On the singularity in Gowdy spacetimes

### **Nonlinear Waves session.**

Serge Alinhac  
University of Paris, Orsay

Title: Energy inequalities and decay for wave equations on a curved background  
Abstract: We display, for various metrics close to Minkowski, energy inequalities with small amplification factors and control of special derivatives. Applications to decay of solutions to wave equations for the corresponding metrics are given. These developments are motivated by applications to quasilinear wave equations.

John Stalker  
Princeton

Title: Scalar waves near Reissner-Nordstrom naked singularities.

Abstract: I will present Strichartz estimates for the scalar wave equation on a Reissner-Nordstrom space-time in the case where the charge is large compared to the mass. In this case there is a naked singularity rather than a black hole. This is joint work with Shadi Tahvildar-Zadeh. The proof is based on recent work, which I will also discuss, on the wave equation in Minkowski space with a singular slowly decaying potential. That work is joint with Nicolas Burq, Fabrice Planchon, and Shadi Tahvildar-Zadeh.

Jacob Sterbenz  
Princeton

Title: Angular regularity and nonlinear wave equations. Strichartz estimates and global existence.

Abstract: We discuss certain improvements to the usual linear and bilinear Strichartz estimates for the wave equation which are based on additional regularity of the initial data with respect to the angular momentum operators  $O_{ij} = (x_i \partial_j - x_j \partial_i)$ . We also discuss how these estimates can be combined with various microlocal decomposition techniques and special function spaces to recover scale-invariant global existence theorems for non-linear wave equations which are not naturally well-posed with respect to translation invariant Sobolev spaces. An example of such equations are the (4+1) dimensional Yang-Mills equations in the Lorentz gauge.

6:00-6:30 Hans Lindblad  
UC San Diego

Title: The weak null condition and global existence for the Einstein equations

## Numerical GR session.

Lee Lindblom

Caltech

Title: Controlling the Growth of Constraints in Hyperbolic Evolution Systems

Abstract: Two methods of limiting the growth of constraints in hyperbolic evolution systems will be discussed: 1) an active control mechanism that monitors the constraints and adjusts the form of the evolution equations whenever necessary to limit their growth; and 2) imposing special boundary conditions on the fields to ensure that constraint violations do not enter the computational domain from the boundaries. The basic ideas for these two methods of controlling the constraints will be discussed, and their effectiveness will be illustrated with examples of 3D numerical solutions of a fairly simple hyperbolic evolution system.

Harald Pfeiffer

Caltech

Title: The initial value problem in numerical relativity

Abstract: We discuss the recent completion of the (York) conformal procedure for the initial value problem of GR. Through a weighted transverse traceless decomposition, complete agreement is achieved between Hamiltonian and Lagrangian formulations of the initial value problem. Furthermore, the procedure is now invariant to conformal transformations of the free data. The plain conformal procedure results in four nonlinear coupled partial differential equations; a slicing condition often adds a fifth elliptic equation for the lapse-function. Although little is known about existence and uniqueness of solutions to these coupled PDE's, we are able to solve them numerically in many cases. We survey such numerical solutions, on maximal as well as non-maximal slices, with and without the assumption of conformal flatness, and with or without inner boundaries representing black holes.

Oscar Reula

Cordoba

Title: Strongly hyperbolic systems in General Relativity

Abstract: We discuss several topics related to the notion of strong hyperbolicity which are of interest in general relativity. After introducing the concept and showing its relevance we provide some covariant definitions of strong hyperbolicity. We then prove that a system is strongly hyperbolic with respect to a given hypersurface, then it is also strongly hyperbolic with respect to any nearby one.

We then study for how much these hypersurfaces can be deformed and discuss then causality, namely what is the maximal propagation speed in any given direction. In contrast with the symmetric hyperbolic case, for which the proof of causality is geometrical and direct, relying in energy estimates, the proof for general strongly hyperbolic systems is indirect for it is based in Holmgren's theorem.

To show that the concept is needed in the area of general relativity we discuss two results for which the theory of symmetric hyperbolic systems shows to be insufficient. The first deals with the hyperbolicity analysis of systems which are second order in space derivatives, they include certain versions of the ADM and the BSSN families of equations. This analysis is considerably simplified by introducing pseudo-differential first order evolution equations. Well posedness for some members of the latter family systems is established by showing they satisfy the

strong hyperbolicity property. Furthermore it is shown that many other systems of such families are only weakly hyperbolic, implying they should not be used for numerical modeling.

The second result deals with systems having constraints. The question posed is which hyperbolicity properties, if any, are inherited from the original evolution system by the subsidiary system satisfied by the constraint quantities. The answer is that, subject to some condition on the constraints, if the evolution system is strongly hyperbolic then the subsidiary system is also strongly hyperbolic and the causality properties of both are identical.

Olivier Sarbach

LSU

Title: Einstein's equations with artificial boundaries.

Abstract: We consider the initial-boundary value problem for symmetric hyperbolic formulations of Einstein's equations in the presence of artificial boundaries which are usually present in three dimensional numerical relativity. Boundary conditions have to be specified such that they are compatible with the constraints, such that they don't introduce too much reflection and such that the resulting initial-boundary value problem is well posed. We discuss such constraint-preserving boundary conditions for several inequivalent formulations of Einstein's equations and also discuss their relevance for numerical simulations.

Steve Liebling

Southampton

Title: Nonlinear Wave Equations and Numerical Relativity

Abstract: Computing dynamic and general solutions to strong field gravity poses a number of very difficult problems, one among these being the issue of computational resources. By considering other less complicated, but still quite interesting, nonlinear models, tools and expertise can be developed and applied in turn to the gravitating case. In particular interesting behavior resembling black hole critical phenomena is observed for the nonlinear sigma model both under assumed spherical symmetry and without that assumption. Preliminary results for certain other models are also presented.

### **Geometric Analysis session.**

Ben Chow

UC San Diego

TBA

David Maxwell

University of Washington

Title: Solutions of the Constraint Equations with Apparent Horizon Boundaries

Piotr Chrusciel

Tours

Title: Initial Data Engineering

Pengzi Miao

MSRI

Title: Ricci Curvature Rigidity for Asymptotically Hyperbolic Manifolds.