

TOPOLOGY SEMINAR SERIES

Wednesday, October 29, 2025

at 10:00 am

Ungar 528-B

BRUNNIAN LINKS AND KONTSEVICH GRAPH COMPLEX

Recently, Tadayuki Watanabe disproved the Smale Conjecture, which stated that the group $\text{Diff}_\partial(D^4)$ is homotopically trivial. He showed that there are nontrivial homotopy groups $\pi_q \text{Diff}_\partial(D^4) \otimes \mathbf{Q}$. The first key idea was to exhibit smooth D^4 -bundles using trivalent graphs equipped with Hopf and Borromean links as their blueprints. The second key in the construction was the use of Kontsevich's configuration space integral to detect the non-triviality of such bundles.

These ideas were generalized by Watanabe for higher-dimensional disks D^d , $d \geq 4$. I will explain some of the ideas and constructions of Watanabe's work. In our very recent joint work, we used Brunnian links to construct a chain map from the Kontsevich graph complex to the rational singular chain complex of $B\text{Diff}_\partial(D^{2k})$ when the dimension $2k$ is sufficiently large. Then we use again Kontsevich's configuration space integral to detect the nontriviality of such homology elements

In particular, we provide new constructions of non-trivial elements in the homotopy groups $\pi_{8k-10}(B\text{Diff}_\partial(D^{2k})) \otimes \mathbf{Q}$ (for $k \geq 17$) which are derived from well-known cycles in the graph complex.